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You teach your children all the time. You taught language to your infants and you read to your son or daughter. You taught them how to count and use basic arithmetic. Here are some ways you can continue to reinforce mathematics learning.

• Encourage a positive attitude toward mathematics.
• Set aside a place and a time for homework.
• Be sure your child understands the importance of mathematics achievement.

The Glencoe Parent and Student Study Guide Workbook is designed to help you support, monitor, and improve your child’s math performance. These worksheets are written so that you do not have to be a mathematician to help your child.

The Parent and Student Study Guide Workbook includes:

• A 1-page worksheet for every lesson in the Student Edition (92 in all). Completing a worksheet with your child will reinforce the concepts and skills your child is learning in math class. Upside-down answers are provided right on the page.

• A 1-page chapter review (12 in all) for each chapter. These worksheets review the skills and concepts needed for success on tests and quizzes. Answers are located on pages 105–108.

Online Resources
For your convenience, these worksheets are also available in a printable format at msmath2.net/parent_student.

Online Study Tools can help your student succeed.

• msmath2.net/extra_examples shows you additional worked-out examples that mimic the ones in the textbook.

• msmath2.net/self_check_quiz provides a self-checking practice quiz for each lesson.

• msmath2.net/vocabulary_review checks your understanding of the terms and definitions used in each chapter.

• msmath2.net/chapter_test allows you to take a self-checking test before the actual test.

• msmath2.net/standardized_test is another way to brush up on your standardized test-taking skills.
You can use a four-step plan to solve problems.

<table>
<thead>
<tr>
<th>Explore</th>
<th>Determine what information is given in the problem and what you need to find. Do you have all the information you need? Is there too much information?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plan</td>
<td>Select a strategy for solving the problem. There may be several strategies that you could use. Estimate the answer.</td>
</tr>
<tr>
<td>Solve</td>
<td>Solve the problem by carrying out your plan. If your plan does not work, try another, and maybe even a third plan.</td>
</tr>
<tr>
<td>Examine</td>
<td>Examine the answer carefully. See if it fits the facts given in the problem. Compare it to your estimate. If your answer is not correct, make a new plan and start again.</td>
</tr>
</tbody>
</table>

**EXAMPLE**

Sergio bought a 30-minute long distance phone card for $4.50. On his home telephone, long distance costs 10 cents per minute. Which is less expensive?

<table>
<thead>
<tr>
<th>Explore</th>
<th>You need to find out whether a long distance call using the calling card or Sergio’s home phone is less expensive.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plan</td>
<td>You need to find the per-minute rate of the calling card and then compare to the per-minute rate on the home phone. Divide the price of the calling card by the number of minutes, and then compare to the home phone rate. You estimate that the home phone is less expensive.</td>
</tr>
<tr>
<td>Solve</td>
<td>$450 \div 30 = 15 \text{ cents per minute. } 10 &lt; 15$</td>
</tr>
<tr>
<td>Examine</td>
<td>The calling card rate is 15 cents per minute, so the home phone rate is less expensive.</td>
</tr>
</tbody>
</table>

**Try This Together**

1. The Washington family is going on a 775-mile vacation. Their car gets 31 miles per gallon of gas. If gas costs $1 per gallon, how much will they spend on gas? *HINT: You need to find the number of gallons of gas the car will use.*

**PRACTICE**

**Use the four-step plan to solve each problem.**

2. **Hobbies** Tristen is making a quilt with his mother. The quilt has a total of 40 squares. Tristen wants to have an equal number of squares with 8 different colors. How many squares of each color will he have?

3. **Standardized Test Practice** The school is buying new risers for the choir to stand on during concerts. There are 120 people in the choir and each riser will hold 20 people. How many risers will they need to buy?
   - A 5
   - B 6
   - C 12
   - D 10

**Answers:** 1. B 2. 5 3. B
Powers and Exponents

When you multiply two or more numbers, each number is called a **factor** of the product. When the same factor is repeated, you can use an exponent to simplify the notation. An **exponent** tells you how many times a number, called the **base**, is used as a factor. A **power** is a number that is expressed using exponents.

<table>
<thead>
<tr>
<th>Examples of Powers</th>
<th>4²</th>
<th>4 × 4</th>
<th>four to the second power, or four squared</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2³</td>
<td>2 × 2 × 2</td>
<td>two to the third power, or two cubed</td>
</tr>
<tr>
<td></td>
<td>5⁴</td>
<td>5 × 5 × 5 × 5</td>
<td>five to the fourth power</td>
</tr>
</tbody>
</table>

### Examples

**A** Write 8 · 8 · 8 in exponential form.

The base is 8. Since 8 is a factor three times, the exponent is 3.

8 · 8 · 8 = 8³

**B** Write 2⁵ as a product, then evaluate.

The base is 2. The exponent 5 means that 2 is a factor five times.

2⁵ = 2 · 2 · 2 · 2 · 2

= 32

### Try These Together

1. Write 14 · 14 · 14 · 14 in exponential form.

   *HINT: How many factors are there?*

2. Evaluate 4³.

   *HINT: The exponent tells how many factors there are.*

### Practice

**Write each power as a product of the same factor.**

3. 3³
4. 6²
5. a⁴
6. b³
7. 4²
8. x⁵
9. 7³
10. 2⁵

**Write each product in exponential form.**

11. 5 · 5 · 5
12. 2 · 2 · 2 · 2 · 2
13. 6 · 6
14. z · z · z · z
15. 8 · 8 · 8 · 8
16. 1 · 1 · 1 · 1 · 1
17. d · d · d
18. 9 · 9

**Evaluate each expression**

19. 8³
20. 12²
21. 3⁵
22. 6⁴
23. 2⁶
24. 10⁴
25. 1⁹
26. 7³

### 27. Standardized Test Practice

Which is equivalent to 5³?

**A** 5 · 5 · 5
**B** 5 + 5 + 5
**C** 3 · 5
**D** 3 · 3 · 3 · 3 · 3
When you evaluate an expression, the **order of operations** ensures that the expression always has only one value. The order of operations tells you which operation to use first.

<table>
<thead>
<tr>
<th>Order of Operations</th>
<th>1. Do all operations within grouping symbols first.</th>
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<td>2. Evaluate all powers before other operations.</td>
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<td>3. Multiply and divide in order from left to right.</td>
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<tr>
<td></td>
<td>4. Add and subtract in order from left to right.</td>
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**Evaluate each expression.**

**A** \[ 15 + 6^2 \div 3 \]

\[ 15 + 6^2 \div 3 = 15 + 36 \div 3 \]

First evaluate 6^2.

\[ = 15 + 12 \]

Second, divide 36 by 3.

\[ = 27 \]

Finally, add 15 and 12.

**B** \[ (15 + 6) \div 3 \]

\[ (15 + 6) \div 3 = 21 \div 3 \]

First add 15 and 6 within the parentheses.

\[ = 7 \]

Second, divide 21 by 3.

**Try These Together**

**Evaluate each expression.**

1. \[ 7 \times 4 + 12 \]

   *HINT: Multiply first.*

2. \[ (11 - 4) \times 3^2 \]

   *HINT: Do operations in parentheses first.*

**Name the operation that should be done first in each expression.**

3. \[ 2 \times 8 + 5 \]

4. \[ 9 - 2^3 \times 4 \]

5. \[ 22 \div (2 + 9) \]

6. \[ (4 - 2) \times 5 \]

**Evaluate each expression**

7. \[ 4^2 \div 2 \times 3 \]

8. \[ (10 + 12) \div 11 \]

9. \[ (15 - 8) \times 3 \]

10. \[ 6^2 \times (9 - 9) \]

11. \[ 12 - 4 \div 2 \]

12. \[ 54 \div 6 + 2^4 \]

13. \[ 24 \div (3 \times 4) \]

14. \[ 7^2 - (2 \times 3) \]

**Insert parentheses to make each sentence true.**

15. \[ 12 + 3 - 1 \times 2 = 16 \]

16. \[ 1 + 8 + 4 \div 2 = 7 \]

17. \[ 16 - 12 \times 3 + 2^3 = 20 \]

18. \[ 7 + 3 \times 8 + 1 = 90 \]

19. **Shopping** Sonny bought 2 comic books that cost $3 each, 5 comic books that cost $2 each and 1 comic book that cost $4. How much did he spend?

20. **Standardized Test Practice** Carlotta scored a 25 on her math test. Her friend scored twice as many points as she did. When Carlotta retook the test, she scored 4 points less than her friend did the first time. Which expression could you use to find Carlotta’s score on her second test?

   **A** \[ 25 - 4 \div 2 \]

   **B** \[ (4 \times 25) \div 2 \]

   **C** \[ (25 \times 2) - 4 \]

   **D** \[ 25 - 4 \times 2 \]
Variables, usually letters, are used to represent numbers in some expressions. The branch of mathematics that involves expressions with variables is called algebra. Algebraic expressions are combinations of variables, numbers, and at least one operation. If you replace variables with numbers, you can evaluate, or find the value of, an algebraic expression.

### Examples

**Evaluate each expression if \( b = 12 \).**

**A** \( 43 - b \)

\[
43 - b = 43 - 12 \\
= 31
\]

Replace \( b \) with 12.

Subtract 12 from 43.

**B** \( 3b + 6 \)

\[
3b + 6 = 3 \times 12 + 6 \\
= 36 + 6 \\
= 42
\]

Replace \( b \) with 12.

Multiply 3 by 12.

Add 36 and 6.

**Try These Together**

**Evaluate each expression if \( r = 8 \) and \( s = 5 \).**

1. \( s + r - 2 \)

HINT: Replace the variables.

2. \( 9r + s \)

HINT: Replace the variables, then multiply.

**Practice**

**Evaluate each expression if \( x = 8 \), \( y = 4 \), and \( z = 2 \).**

3. \( x - y \)

4. \( y + z \)

5. \( x \div y \)

6. \( x + y + z \)

7. \( y \div z \)

8. \( xy \)

9. \( yz - x \)

10. \( xz \div 2 \)

11. \( 2y - 3z \)

12. \( 3x - 10 \)

13. \( 3yz \)

14. \( x + y - 2z \)

15. Evaluate \( 20x + 3x \) if \( x = 6 \).

16. **Business** Gerod grows tomatoes on his family’s farm and sells them at the market every Saturday. He earns $2 for every pound of tomatoes. Write an algebraic expression to show how much money he earns for \( n \) pounds of tomatoes.

17. **Standardized Test Practice** Marco works at a car wash in the summer. He earns $2 for each car he washes and $3 for each car he vacuums. The amount of money he earns is represented by the expression \( 2w + 3v \). If he washes 10 cars and vacuums 20 cars, how much money will he earn?

A $70  
B $80  
C $50  
D $90
In mathematics, an **equation** is a sentence that contains an equals sign, \(=\). You **solve** the equation when you replace the variable with a number that makes the equation true. Any number that makes the equation true is called a **solution**. When you write an equation that represents a real-world problem, you are **modeling** the problem.

**EXAMPLES**

A Solve \(y + 7 = 10\) mentally.
\[
\begin{align*}
3 + 7 & \leq 10 \quad \text{You know that } 3 + 7 = 10. \\
10 & = 10 \quad \checkmark
\end{align*}
\]
The solution is 3.

B Solve \(5a = 35\) mentally.
\[
\begin{align*}
5(7) & = 35 \quad \text{You know that } 5(7) = 35. \\
35 & = 35 \quad \checkmark
\end{align*}
\]
The solution is 7.

**Try These Together**

**Solve each equation.**

1. \(s + 9 = 22\)  
   **HINT:** What plus 9 equals 22?

2. \(13n = 39\)  
   **HINT:** 13 times what equals 39?

**Name the number that is the solution of the given equation.**

3. \(17 - x = 15\); 2, 3, 4

4. \(12 + y = 17\); 3, 4, 5

5. \(2 + z = 10\); 7, 8, 9

6. \(m + 5 = 10\); 4, 5, 6

7. \(15 \div n = 3\); 3, 4, 5

8. \(2j = 6\); 1, 2, 3

**Solve each equation.**

9. \(a + 5 = 11\)

10. \(10 - b = 2\)

11. \(4 + w = 25\)

12. \(p - 30 = 10\)

13. \(q = 3 + 6\)

14. \(r = 2(9)\)

15. \(4s = 8\)

16. \(9 - t = 2\)

17. \(24 \div f = 6\)

18. \(3g = 36\)

19. \(h + 23 = 33\)

20. \(j = 5 - 2\)

21. **Food** If Deepak drinks 28 glasses of milk every week, what is the average number of glasses of milk he drinks each day? Use the equation \(28 = 7m\), where \(m\) is the number of glasses of milk per day.

22. **Standardized Test Practice** Gabriel has 20 minutes to take a math quiz. The quiz has 10 problems on it. Which equation shows how to find how many minutes Gabriel can spend on each problem?

   A \(20 \times 10 = p\)  
   B \(p = 10 \div 20\)  
   C \(p = 20 + 10\)  
   D \(20 \div 10 = p\)
Properties (pages 30–33)

In algebra, **properties** are statements that are true for any number. They often provide a method for writing equivalent expressions. For example, the expressions $4(9 + 2)$ and $4(9) + 4(2)$ are **equivalent expressions** because they have the same value, 44. This shows how the **Distributive Property** combines addition and multiplication.

<table>
<thead>
<tr>
<th>Commutative Property</th>
<th>$a + b = b + a$</th>
<th>$a \times b = b \times a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Associative Property</td>
<td>$(a + b) + c = a + (b + c)$</td>
<td>$(a \times b) \times c = a \times (b \times c)$</td>
</tr>
<tr>
<td>Identity Property</td>
<td>$a + 0 = a$</td>
<td>$a \times 1 = a$</td>
</tr>
<tr>
<td>Distributive Property</td>
<td>$a(b + c) = a(b) + a(c)$</td>
<td></td>
</tr>
</tbody>
</table>

**EXAMPLES**

A Evaluate $3(4 + 8)$.

$3(4 + 8) = 3 \cdot 4 + 3 \cdot 8$

**Rewrite the expression using the Distributive Property.**

Multiply

$= 12 + 24$

Add.

$= 36$

B Use the Associative Property to write an expression equivalent to $(5 + 6) + 2$.

$(5 + 6) + 2 = 5 + (6 + 2)$

**Rewrite the expression using the Associative Property.**

Add.

$11 + 2 = 5 + 8$

Add. The expressions are equivalent.

$13 = 13$

**Try These Together**

**Name the property shown.**

1. $\left( \frac{2}{5} + \frac{4}{5} \right) + \frac{3}{8} = \frac{2}{5} + \left( \frac{4}{5} + \frac{3}{8} \right)$

**HINT:** Notice that the grouping or associating is changed.

2. $\frac{5}{9} \times 1 = \frac{5}{9}$

**HINT:** Notice that a fraction is multiplied by 1.

**PRACTICE**

**Name the property shown by each statement.**

3. $1.45 \times 1 = 1.45$

4. $(1 \times 2) \times 7 = 1 \times (2 \times 7)$

5. $9(6 + 4) = 9 \cdot 6 + 9 \cdot 4$

6. $6 + 3 = 3 + 6$

7. **Standardized Test Practice**

Name the property shown by this statement.

$\frac{7}{11} \times \left( \frac{4}{9} + \frac{2}{3} \right) = \frac{7}{11} \times \frac{4}{9} + \frac{7}{11} \times \frac{2}{3}$

A identity  B associative  C commutative  D distributive


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Parent and Student Study Guide

Mathematics: Applications and Concepts, Course 2
A sequence of numbers is a list in a specific order. The numbers in a sequence are called terms. If you can always add the same number to the previous term to find the next term, the sequence is an arithmetic sequence. If you can always multiply the previous term by the same number to find the next term, the sequence is a geometric sequence.

Finding the Pattern in a Sequence

- To test for an arithmetic sequence, try subtracting the first term from the second. Then test to see if this same number separates each term of the sequence.
- To test for a geometric sequence, try dividing the second term by the first. Then test to see if this same quotient multiplies each term to give the next term in the sequence.

EXAMPLES

A What is the pattern in this sequence? 
1, 2, 3, 4, 5, …
Is it arithmetic, geometric, or neither?
The pattern is that the first number, and every other number, is 1 and the numbers between are the counting numbers, starting with 2. This sequence is neither arithmetic nor geometric.

B What is the pattern in this sequence?
0, 3, 6, 9, 12, 15, …
What is the next number?
Is this sequence arithmetic, geometric, or neither?
Each term is equal to the previous term plus 3.
The next number is 18. This is an arithmetic sequence.

Try These Together

1. Describe the pattern in 1, 3, 5, 7, … and find the next three terms. Identify the sequence as arithmetic, geometric, or neither. 
HINT: Can you add the same number to 1 to get 3 as you can add to 3 to get 5?

2. Give four terms of a sequence with this rule: Begin with 1 and multiply each term by 5. 
HINT: The first term is 1 and the second is 5.

PRACTICE

Describe the pattern in each sequence. Identify the sequence as arithmetic, geometric, or neither. Then find the next three terms.

3. 10, 20, 40, 80, 160, …
4. 0, 1, 3, 6, 10, …
5. 30, 33, 36, 39, 42, …

Create a sequence using each rule and give four terms beginning with the given number. State whether the sequence is arithmetic, geometric, or neither.

6. Add 4 to each term; 8.
7. Add 2 to each term; 50.

8. Standardized Test Practice What is the missing term in this sequence?
6, 12, 18, ___, 30, 36, 42, …

A 24
B 20
C 23
D 19
The following table describes the basic units of measurement in the metric system.

<table>
<thead>
<tr>
<th>Length</th>
<th>The metric unit of length is the <strong>meter (m)</strong>. A meter is about the distance from the floor to a doorknob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>The metric unit of mass is the <strong>kilogram (kg)</strong>. Mass is the amount of matter that an object contains. Your math textbook has a mass of about one kilogram.</td>
</tr>
<tr>
<td>Capacity</td>
<td>The <strong>liter (L)</strong> is the basic unit of capacity in the metric system. Capacity is the amount of dry or liquid material an object can hold. Soft drinks often come in 2-liter plastic containers.</td>
</tr>
</tbody>
</table>

The basic metric units can be changed into larger or smaller units by dividing or multiplying by powers of 10. For example, 1 kilometer is $1 \times 10^3$ m. The chart below shows the relationship between the metric units and the powers of 10.

![Metric Units Chart]

To change from a larger unit to a smaller unit, you need to multiply. To change from a smaller unit to a larger unit, you need to divide.

<table>
<thead>
<tr>
<th>MULTIPLY</th>
<th>1,000</th>
<th>100</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>km</td>
<td>m</td>
<td>cm</td>
<td>mm</td>
</tr>
<tr>
<td>1,000</td>
<td>100</td>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

**EXAMPLES**

**A** 4.6 L = ? mL
To change from liters to milliliters, multiply by 1,000 since 1 L = 1,000 mL.
4.6 $\times$ 1,000 = 4,600
4.6 L = 4,600 mL

**B** 122 cm = ? m
To change from centimeters to meters, divide by 100 since 1 m = 100 cm.
122 $\div$ 100 = 1.22
122 cm = 1.22 m

**PRACTICE**

**Complete.**
1. 5 m = ? cm
2. 96 cm = ? mm
3. 150 mm = ? cm
4. 2.5 kL = ? L
5. 1,200 g = ? kg
6. 1,565 mL = ? L

**7. Standardized Test Practice** There are 8,000 milligrams of protein in one serving of peanut butter. How many grams of protein are there in 5 servings of peanut butter?

A 0.4 g  B 40 g  C 4 g  D 400 g

Answers: B 0.4 g  B 40 g  C 4 g  D 400 g
Scientific Notation (pages 43–45)

You can write numbers such as 4.5 billion in scientific notation by using a power of ten.

<table>
<thead>
<tr>
<th>Scientific Notation</th>
<th>Numbers expressed in scientific notation are written as the product of a number that is at least one but less than ten and a power of ten. The power of ten is written with an exponent. To write a number in scientific notation, move the decimal point to the right of the first nonzero digit, and multiply this number by a power of ten. To find the power of ten, count the number of places you moved the decimal point. The decimal part of a number written in scientific notation is often rounded to the hundredths place.</th>
</tr>
</thead>
</table>

**EXAMPLES**

A Write 45,692 in scientific notation.

- \(4.5692\) Move the decimal point 4 places to get a number between 1 and 10.
- \(4.5692 \times 10^4\) You moved the decimal point 4 places so the power of ten is 4.
- \(4.57 \times 10^4\) Round to the nearest hundredth.

B Write \(4.5 \times 10^9\) in standard form.

- \(10^9 = 1,000,000,000\)
- \(4.5 \times 10^9 = 4.5 \times 1,000,000,000\)
- Notice the decimal point moved 9 places to the right.

Try These Together

1. Write 734 in scientific notation.

   **HINT:** Move the decimal point to the right of the 7. Count the number of places you moved the decimal point.

2. Write \(9.3 \times 10^7\) in standard form.

   **HINT:** The power of 10 indicates the number of places the decimal moves.

**PRACTICE**

Write numbers already in standard form in scientific notation, and numbers already in scientific notation, write in standard form.

3. 650
4. 5,000
5. \(8.5 \times 10^3\)
6. \(1.5 \times 10^6\)
7. \(6.07 \times 10^3\)
8. 640,000
9. 3,300
10. 28,000,000
11. \(3.0 \times 10^2\)
12. 6,000 \(\bullet\) \(6 \times 10^2\)
13. 1,200 \(\bullet\) \(1.2 \times 10^4\)
14. 30,500 \(\bullet\) \(3.05 \times 10^2\)

Replace each \(\bullet\) with \(<, >,\) or \(=\) to make a true sentence.

15. **Money Matters** The national debt of a country is the amount of money it has borrowed from its people or other countries. In 1919, the national debt of the United States was \(25.5\) billion dollars. Write \(25.5\) billion in scientific notation.

16. **Standardized Test Practice** Comets are clumps of dust and frozen gases floating in the solar system. Recently, pieces of a comet slammed into Jupiter’s atmosphere at \(210,000\) km/h. Write \(210,000\) in scientific notation.

   A \(2.1 \times 10^4\)
   B \(2.1 \times 10^2\)
   C \(2.1 \times 10^3\)
   D \(2.1 \times 10^5\)
Chapter 1 Review

Treasure Hunt
For the Math Club party, Mitch plans a treasure hunt for the members. Each clue is a math problem. All of the clues together spell out the name of the treasure.

Find each clue.
1. One step in the four-step problem-solving plan involves looking at your answer carefully and seeing if it fits the facts in the problem. What is the name of this step? For Clue 1, use the first letter of this word.

2. Evaluate this expression using the order of operations.
   \[20 \div 4 + 1(6 - 1) + 3(4)\]

For Clues 3–6, find the value of each expression.
3. \[6^2 + 2^2 - 5(4)\]
4. \[2^4 - 3\]
5. \[5(t - s) + r\] if \(t = 7, s = 6,\) and \(r = 4\)
6. \[3^3 - 4^2\]

7. Solve the equation \(\frac{t}{5} = 3\) mentally.

8. Find the next number in this pattern.
   \[1\ 2\ 3\ 1\ 1\ 2\ 2\ 3\ 3\ 1\ 1\ 1\ 2\ 2\ 2\]

To discover what the treasure is, make each numbered clue from 2 to 26 into a letter by using the corresponding letter of the alphabet, so 2 = B, 3 = C, and so on, down to 26 = Z. Remember that you found the letter for Clue number 1 in Exercise 1 above.

Write the letters in the blanks that correspond to the numbers of the clues to read the name of the treasure.

Clue: 4 7 2 5 1 3 5 8 6 1 3

Answers are located on page 105.
Frequency Tables (pages 54–57)

You can organize large amounts of data in a frequency table, which shows the number of times each item appears. A frequency table has a scale that includes all of the numbers in the data. A frequency table also has an interval, which separates the scale into equal parts.

**Example**

Luis surveyed T-shirt shops in Port Aransas to find the average prices (in dollars) of souvenir T-shirts. The average prices he found at each shop were: 9, 10, 12, 9, 18, 12, 13, 10, 5, 8, 16, and 11. Create a frequency table with these data.

- The frequency table includes all of the data, so the scale will be from 1–20.
- The interval, which divides the data into four equal parts, is 5.

### Try This Together

1. Every student in the Pet Lovers Club has at least one pet. The number of pets each member has is 1, 2, 1, 1, 3, 2, 4, and 6. Make a frequency table with this data. Identify the scale and interval.

   **Hint:** The scale must include all of the data, and the interval must divide the data into equal-sized parts.

### Choose an appropriate scale and interval to make a frequency table for each set of data.

2. 5, 3, 2, 1, 4, 6, 9
3. 15, 10, 24, 20, 37, 40
4. 111, 125, 101, 94, 136
5. 220, 340, 130, 180, 230, 340, 100
6. 5, 6, 11, 0, 14, 12, 8, 16, 18
7. 20, 10, 50, 60, 40, 30, 90, 70
8. Draw a number line that shows a scale of 0 to 10 and an interval of 2.

9. Food  Alex’s class is having a pizza party. The set of data shows what kinds of pizza people ordered. Make a frequency table of the data.

### Standardized Test Practice

What is an appropriate scale for this set of data?

15, 10, 12, 16, 18, 5, 3, 46, 35, 21, 26

A 5–50  B 0–45  C 3–45  D 1–50

Kind of Pizza

<table>
<thead>
<tr>
<th></th>
<th>S</th>
<th>P</th>
<th>V</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pizza</td>
<td>S</td>
<td>P</td>
<td>M</td>
<td>V</td>
</tr>
</tbody>
</table>

S = sausage
P = pepperoni
M = mushroom
V = vegetable

Answers: 1. See Answer Key 2. 7. 0–99, 20 8. See Answer Key 10. D 5. 100–349, 50 6. 0–19 2. 0–99, 20 8. See Answer Key 10. D 5. 100–349, 50
Making Predictions

The following examples show a line graph and a scatter plot.

**Examples**

A Line Graph

B Scatter Plot

You can make predictions from a line graph because it shows a trend over time. You can also make predictions from a scatter plot based on the relationship it shows.

**Try This Together**

1. The line graph shows how the population of San Diego has grown since 1960. Predict the population of San Diego in the year 2010. 
   Hint: Where do you expect to see the line in the year 2010?

2. Money Matters The line graph shows how much allowance Marte receives every week. Predict how much allowance she will receive every week when she is 13 years old.

3. Standardized Test Practice The scatter plot shows the number of customers at an ice cream shop and the high temperature for that day. Predict the number of customers when the temperature reaches 85°F.

**Answers:**
1. Sample answer: 1.5 million
2. $6 per week
3. C 110

A line plot is a picture of information on a number line. To make a line plot, determine the scale and interval from your data, just as you do when you make a frequency table. Then draw the number line to fit the scale and mark the interval. Finally, place an “X” above the number line to represent each data point.

**Example**

For one week, Juko kept track of how many glasses of water she drank each day. Here are the results: 2, 5, 3, 5, 7, 4, and 5. Create a line plot with these data.

The scale must include all of the numbers in the data set, so you can choose 1–8. You can choose an interval of 1. Notice how the line plot shows a group of marks at 5. Data that are grouped closely together are called a **cluster**.

**Try These Together**

Make a line plot for each set of data.

1. 1, 5, 6, 3, 3, 8, 4, 5, 6  
2. 30, 25, 60, 45, 60, 25, 45

Hint: Each scale must include all of the numbers in the data set.

**Make a line plot for each set of data.**

4. 32, 35, 32, 32, 31, 36, 38, 38, 39

5. 82, 85, 86, 95, 84, 96, 95, 84  
6. 2, 6, 5, 8, 6, 3, 1, 9, 4

7. Nutrition  The data at the right shows the daily recommended number of servings from each food group. Draw a line plot of these data.

<table>
<thead>
<tr>
<th>Food Group</th>
<th>Minimum Number of Servings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dairy Products</td>
<td>2</td>
</tr>
<tr>
<td>Meat, Fish, and Eggs</td>
<td>2</td>
</tr>
<tr>
<td>Vegetables</td>
<td>3</td>
</tr>
<tr>
<td>Fruits</td>
<td>2</td>
</tr>
<tr>
<td>Bread and Grains</td>
<td>6</td>
</tr>
</tbody>
</table>

8. **Standardized Test Practice**  Look at the line plot at the right. Where do you find a cluster?

A 0–5  
B 6–10  
C 11–15  
D 16–20

Answers: 1–7. See Answer Key.

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Mean, Median, and Mode

You can summarize a set of data with a single number in three ways: the mean, median, and mode.

| Mean | The mean of a data set is the arithmetic average. To find the mean, add all of the numbers in the data set and then divide by the number of items in the set. |
| Mode | The mode of a data set is the number(s) or item(s) that appear most often. Sometimes there is no mode, or two or more modes. |
| Median | The median is the middle number in a data set when the data are arranged in numerical order. When there is an even number of items in a data set, the median is the mean of the two middle numbers. |

**Example**

Find the mean, mode, and median for 1, 3, 3, 4, 6, 6, 6, 8, and 8.

To find the mean, you must average the numbers.

\[ \frac{1 + 3 + 3 + 4 + 6 + 6 + 6 + 8 + 8}{9} = 5 \]

To find the mode, look for the number that appears most often.

The number 6 appears most often, so the mode is 6.

The median is the middle number in the data set.

The data are already arranged in numerical order. There are 9 items in the set, so the fifth one, or 6, is the median.

**Try These Together**

Find the mean, mode(s), and median for each set of data.

1. 17, 15, 15, 12, 16, 18, 19

2. 1, 5, 8, 9, 5, 6, 9, 5, 2, 10

3. 46, 52, 23, 28, 32, 25, 23, 51

4. 106, 180, 152, 148, 132, 152

5. 21, 22, 23, 25, 23, 24, 23

6. 200, 350, 375, 425, 200, 250

7. Number of Pets per Student

<table>
<thead>
<tr>
<th>Number of Pets</th>
<th>Tally</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

8. Scores on Spelling Quiz

9. **Standardized Test Practice**

The low temperatures in San Diego for one week in the winter were 55°, 52°, 59°, 59°, 53°, and 52° F. What was the mean low temperature?

A 55°  B 56°  C 58°  D 59°
You can use a stem-and-leaf plot to display data. In a stem-and-leaf plot, the last digit in each data item is the leaf. The digits in front of the leaf become the stem.

**EXAMPLES**

Create stem-and-leaf plots with the following data.

A  Gladys has four dogs. They weigh 42, 58, 53, and 61 pounds.
Use the tens digits to form the stems, and the ones digits to form the leaves. So 4|2 = 42.
There are two values, 53 and 58, that have a five in the tens place. This causes the 5 stem to have leaves of 3 and 8, to represent 53 and 58.

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>3, 8</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
</tr>
</tbody>
</table>

B  Gladys also has three cats. They weigh 10, 13, and 9 pounds.
Use the tens digits to form the stems, and the ones digits to form the leaves. So 0|9 = 9.

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>9</td>
</tr>
<tr>
<td>1</td>
<td>0, 3</td>
</tr>
</tbody>
</table>

Try These Together

Make a stem-and-leaf plot for each data set.

1. 11, 5, 6, 12, 23, 24, 28, 21, 18, 17
2. 36, 41, 25, 28, 30, 45, 45, 40, 26, 29

3. 15, 13, 26, 12, 14, 23, 26, 21, 15
4. 92, 86, 85, 66, 73, 72, 64, 75, 84, 81
5. 2, 6, 3, 5, 11, 15, 16, 18, 7, 9, 19
6. 56, 54, 28, 41, 33, 26, 58, 64, 24, 45

7. Music  The data table lists prices of compact discs (CDs) in two different music categories at Annie’s Music Shop. Make a stem-and-leaf plot of the prices of classical music compact discs and country music compact discs. Which category of music CDs is priced lower at Annie’s Music Shop?

<table>
<thead>
<tr>
<th>Type of Music CD</th>
<th>Prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Classical</td>
<td>8, 7, 15, 13, 13, 12, 10, 14, 16, 11, 13, 15, 9</td>
</tr>
<tr>
<td>Country</td>
<td>12, 9, 14, 15, 16, 17, 14, 15, 12, 11, 9, 9</td>
</tr>
</tbody>
</table>

8. Standardized Test Practice  Look at the stem-and-leaf plot of prices of tickets to a play. In which interval do the greatest number of prices fall?

- A 1–10
- B 32–38
- C 11–19
- D 20–29

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1, 3, 5, 7, 9</td>
</tr>
<tr>
<td>2</td>
<td>0, 5, 7, 9</td>
</tr>
<tr>
<td>3</td>
<td>2, 4, 6, 312 = 32</td>
</tr>
</tbody>
</table>

Answers 1–7. See Answer Key.
You can use a box-and-whisker plot to summarize data. A box-and-whisker plot divides a data set into four parts.

**Example**

Create a box-and-whisker plot for these data:
12, 16, 62, 48, 16, 59, 43, 39.

1. Write the data in order from least to greatest. Find the median of the data, which is 41.
2. Find the lower quartile, which is the median of the lower half of the data, or 16.
3. Find the upper quartile, which is the median of the upper half of the data, or 53.5.
4. The lower extreme is the least value. The upper extreme is the greatest value. The lower extreme is 12 and the upper extreme is 62.

**Step 1**

- Draw a number line. Graph the values you found in step 1.
- Draw a box around the lower and upper quartiles and draw a vertical line through the median value.
- Extend whiskers from each quartile to the extreme points.

**Step 2**

- Calculate the interquartile range by subtracting the lower quartile from the upper quartile. The interquartile range for these data is 37.5.
- Data that are more than 1.5 times the interquartile range from the quartiles are outliers. The limits on outliers are the points beyond which data are considered to be outliers. There are no outliers for these data.

**Step 3**

**Practice**

Use the box-and-whisker plot of test scores.

1. What is the median?
2. What is the lower quartile?
3. What is the upper quartile?
4. What is the lower extreme?
5. What is the upper extreme?
6. What is the interquartile range?
7. What are the limits on outliers?
8. Are there any outliers?

9. **Sports** The table shows the games won by each girls’ sports team at Dennison High School. Make a box-and-whisker plot of the data.

<table>
<thead>
<tr>
<th>Sport</th>
<th>Wins</th>
<th>Sport</th>
<th>Wins</th>
<th>Sport</th>
<th>Wins</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soccer</td>
<td>5</td>
<td>Volleyball</td>
<td>6</td>
<td>Hockey</td>
<td>4</td>
</tr>
<tr>
<td>Basketball</td>
<td>7</td>
<td>Softball</td>
<td>7</td>
<td>Kickball</td>
<td>8</td>
</tr>
</tbody>
</table>

10. **Standardized Test Practice** What is the upper extreme of this box-and-whisker plot?

A 5  B 4  C 3  D 9

A **bar graph** is one method of comparing data by using solid bars to represent quantities. A special kind of bar graph, called a **histogram**, uses bars to represent the frequency of numerical data that have been organized in intervals. Use a bar graph to show the value of different items or categories. Use a histogram to show how many items are contained in each interval.

**Example**

Determine which graph is a histogram and which is a bar graph.

**Graph A**

- **Test Grades**
- **Grade**
- **Students**

**Graph B**

- **Test Grades**
- **Grade (percent)**
- **Frequency**

**Graph A** is a bar graph because it shows the values of five different categories, which are letter grades. **Graph B** is a histogram because it shows the number of students receiving the scores described by each interval.

**Try This Together**

1. Members of the student council cannot agree on which day of the week to have their meeting. Refer to the set of data that shows how many students voted for each day. Determine whether a bar graph or a histogram should be used to show the data then draw the graph.

   **HINT:** Does the data require intervals?

<table>
<thead>
<tr>
<th>Day</th>
<th>Number of Votes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday</td>
<td>4</td>
</tr>
<tr>
<td>Tuesday</td>
<td>3</td>
</tr>
<tr>
<td>Wednesday</td>
<td>5</td>
</tr>
<tr>
<td>Thursday</td>
<td>6</td>
</tr>
<tr>
<td>Friday</td>
<td>2</td>
</tr>
</tbody>
</table>

2. Make a histogram of the data below.

<table>
<thead>
<tr>
<th>Points Scored by the Football Team</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
</tr>
<tr>
<td>31</td>
</tr>
<tr>
<td>28</td>
</tr>
</tbody>
</table>

3. Make a bar graph of the data below.

<table>
<thead>
<tr>
<th>Bake Sale Item</th>
<th>Number Sold</th>
</tr>
</thead>
<tbody>
<tr>
<td>cookie</td>
<td>45</td>
</tr>
<tr>
<td>brownie</td>
<td>37</td>
</tr>
<tr>
<td>cupcake</td>
<td>35</td>
</tr>
<tr>
<td>pie</td>
<td>20</td>
</tr>
<tr>
<td>cheesecake</td>
<td>30</td>
</tr>
</tbody>
</table>

**Standardized Test Practice**

4. Which is **not** a true statement?
   - A A histogram shows data organized by intervals.
   - B Bars on a histogram may be different widths.
   - C A bar graph shows the value of different categories.
   - D The number of wrestlers in each weight group could be displayed on a histogram.

   **Answers:** 1. bar graph; See answer key. 2-3. See answer key.
Statistics and graphs can be presented in ways that are misleading. For example, the mean is not a good way to describe a data set when there are outliers. Changing the scale of a graph may also make the graph misleading.

**EXAMPLE**

Darnell was notified that the raffle ticket he bought at the county fair was a winner. He was told that the average prize amount was almost $4,000. Use the information at the right to explain why Darnell should not expect to win more than $100.

There are only three prizes over $100, and 50 prizes of $100. Because of the large number of $100 prizes, Darnell is most likely to win only $100.

<table>
<thead>
<tr>
<th>Raffle Prizes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prize</td>
</tr>
<tr>
<td>Grand Prize</td>
</tr>
<tr>
<td>First Prize (2 winners)</td>
</tr>
<tr>
<td>Second Prize (50 winners)</td>
</tr>
</tbody>
</table>

The average winner wins almost $4,000!

**Try This Together**

1. Abigail’s starting salary was $47,000, three years ago. She has received a $2,000 raise each year. Create a graph that makes her salary increases look more substantial than they actually are.

2. The two line graphs show sales of T-shirts at The Tee Shop for May. Which graph could be misleading? Explain.

3. **Standardized Test Practice** The Italian Restaurant advertises huge meals. The sizes of many of their meals are shown in the table at the right. What misleading statistic might they be using to describe the serving sizes of their meals?
   - A mean
   - B median
   - C mode
   - D distorted facts

<table>
<thead>
<tr>
<th>Meal</th>
<th>Serving Size (oz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spaghetti</td>
<td>10</td>
</tr>
<tr>
<td>Tortellini</td>
<td>12</td>
</tr>
<tr>
<td>Manicotti</td>
<td>14</td>
</tr>
<tr>
<td>Ravioli</td>
<td>11</td>
</tr>
<tr>
<td>Lasagna</td>
<td>15</td>
</tr>
<tr>
<td>Fettucine</td>
<td>32</td>
</tr>
</tbody>
</table>
Chapter 2 Review

Safe Combinations

To find out what is in a secret safe, you must find the combination to the two locks on the safe. Use the stem-and-leaf plot of midterm-exam scores to answer the questions below to find the combinations for the girls’ lock and the boys’ lock.

1. How many girls are there in the class?
2. What was the lowest girls’ score?
3. What was the highest girls’ score?

Use the answers for Exercises 1–3 to fill in the combination for the girls’ lock above.

4. How many boys are there in the class?
5. What was the highest boys’ score?
6. What was the lowest boys’ score?

Use the answers for Exercises 4–6 to fill in the combination for the boys’ lock above.

Answers are located on page 106.
Integers and Absolute Value (pages 106–108)

An integer is any number from the set \{\ldots, -3, -2, -1, 0, +1, +2, +3, \ldots\}.
Integers greater than 0 are positive integers. Integers less than 0 are negative integers. Zero is neither positive nor negative.

- The absolute value of an integer is its distance from 0 on a number line. The absolute value of \(n\) is written as \(|n|\).
- Numbers that are the same distance from 0, but on opposite sides of 0, have the same absolute value. For example, the absolute value of both \(-3\) and 3 is 3.

**EXAMPLES**

A  Find the absolute value of 6.

On the number line, 6 is 6 units from 0, so \(|6| = 6\).

B  Find the absolute value of \(-4\).

On the number line, \(-4\) is 4 units from 0, so \(|-4| = 4\).

**Try These Together**

1. Write an integer to represent a debt of $6.
   \(\text{HINT: A debt can be written as a negative integer.}\)

2. Find \(|y|\) if \(y = -42\).
   \(\text{HINT: What is the distance from -42 to zero, without regard to direction?}\)

**PRACTICE**

Write an integer for each situation.

3. to move back 3 spaces
4. \(20^\circ\)F below zero
5. a loss of 15 yards
6. a shirt that shrunk 4 inches

Write the integer represented by the point for each letter. Then find its absolute value.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-10</td>
<td>-8</td>
<td>-6</td>
<td>-4</td>
<td>-2</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

13. **Sports**  Steve Cram was one of the world’s top runners in the 1980s.
When he was 17 years old, he ran a mile in 3 minutes and 57 seconds.
When he broke the world record in 1985, he ran about 11 seconds faster. Express this lowering of his time as an integer.

14. **Standardized Test Practice**  Maggie jumped 4 feet from her trampoline down to the ground. Which integer describes the change in her distance from the ground?

A 1  B –1  C –4  D 4

Answers:

Comparing and Ordering Integers
(pages 109–111)

You can use a number line to order integers.

**Ordering Integers**

On a number line, a number to the left is less than a number to the right.

**EXAMPLES**

**A** Which is greater, −4 or 2?

On a number line, −4 is to the left of 2, so −4 < 2 or 2 > −4.

**B** Order these integers from least to greatest. 3, 0, −5, −1, 2, −3, −6

Think of these numbers on a number line in order from left (least) to right (greatest).

−6, −5, −3, −1, 0, 2, 3

---

**Try These Together**

*Replace each ● with < or > to make a true sentence.*

1. −12 ● −6
   
   **HINT:** Which integer is to the left on a number line?

2. 8 ● −9.
   
   **HINT:** A positive integer is always greater than a negative integer.

---

**PRACTICE**

*Replace each ● with < or > to make a true sentence.*

3. −5 ● −6
4. 15 ● −2
5. 17 ● −18
6. 25 ● 28
7. −16 ● −28
8. −2 ● −8
9. −19 ● 19
10. 30 ● 26
11. −19 ● 21
12. −45 ● −43

---

Order the integers from least to greatest.

13. 8, −3, 6, −4, 5
14. 17, 12, −14, −6, 5, −3, −2

15. Which is greater, 8 or −8?

16. **Weather** The high temperatures for one week in Minneapolis, Minnesota, were 0°, −5°, −2°, 3°, 8°, 10°, and −16° Fahrenheit. Order the temperatures from least to greatest.

17. **Standardized Test Practice** Order the integers, −7, 8, −11, and 14 from greatest to least.

   **A** 14, 8, −7, −11
   **B** −7, 8, −11, 14
   **C** −11, −7, 8, 14
   **D** −7, −11, 8, 14

---

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Mathematics: Applications and Concepts, Course 2
You can graph points in a plane on a coordinate system.

A coordinate system has a horizontal number line (called the \(x\)-axis) and a vertical number line (called the \(y\)-axis). These lines cross at right angles at a point called the origin. These lines separate the plane into four quadrants. You can name any point on a coordinate system using an ordered pair of numbers. The first number in an ordered pair is the \(x\)-coordinate. It tells how far the point is to the right or left of the origin. The second number in an ordered pair is the \(y\)-coordinate. It tells how far the point is up or down from the origin.

**Examples**

A The point at \((4, -3)\) is in which quadrant?

The first number in the ordered pair (4), tells you to move 4 units to the right from the origin. The second number \((-3)\) tells you then to move down 3 units from there. The point at \((4, -3)\) is in Quadrant IV.

B In which quadrant are both the coordinates negative?

In Quadrant III, the points are to the left and down from the origin. Both coordinates of the ordered pairs in Quadrant III are negative.

**Try These Together**

1. In the graph below, what is the ordered pair for point \(K\)? In which quadrant is \(K\)?

   \(HINT: \) How far to the left of the origin is \(K\)? How far down?

2. In the graph below, what is the \(x\)-coordinate of point \(B\)? What is the \(y\)-coordinate of \(B\)?

   \(HINT: \) \(B\) is in Quadrant I, so both coordinates are positive.

**Practice**

Name the ordered pair for each point labeled at the right.

3. \(D\)
4. \(L\)
5. \(A\)
6. \(G\)
7. \(C\)
8. \(M\)
9. \(N\)
10. \(E\)

11. **Standardized Test Practice** In which quadrant of a coordinate plane would you find \(F(-4, -1)\)?

   \(A\) Quadrant I \(B\) Quadrant II \(C\) Quadrant III \(D\) Quadrant IV

   \(\)
Two integers that are opposites are called **additive inverses**. The Additive Inverse Property states that the sum of any number and its additive inverse is 0.

\[ 3 + (-3) = 0 \quad a + (-a) = 0 \]

**Adding Integers**

- **Adding Integers with the Same Sign**
  - The sum of two positive integers is positive.
  - The sum of two negative integers is negative.

- **Adding Integers with Different Signs**
  - To add integers with different signs, subtract their absolute values. The sum is
    - positive if the positive integer has the greater absolute value.
    - negative if the negative integer has the greater absolute value.

**Examples**

**A** Solve \( a = -7 + 3 \).

*The signs of these two integers are different.*

Find the absolute value of each.

\[ |-7| = 7, \quad |3| = 3 \]

Notice that \(-7\) has the greater absolute value.

Subtract the absolute values: \(7 - 3 = 4\).

The sum of \(-7 + 3\) is \(-4\). The sum has the sign of the integer that has the greater absolute value.

**B** Evaluate \( x + (-3) \) if \( x = -2 \).

*The signs are the same.*

\( x + (-3) = -2 + (-3) \) Replace \( x \) with \(-2\).

The sum of two negative integers is negative.

\( -2 + (-3) = -5 \)

**Try These Together**

1. Is \(-4 + (-8)\) positive, negative, or 0? **HINT:** Examine the signs.
2. Is \( 7 + (-2)\) positive, negative, or 0? **HINT:** Which number has the greater absolute value?

**Practice**

**Tell whether the sum is positive, negative, or zero.**

3. \(-10 + 12\)  
4. \(-7 + 7\)  
5. \(-6 + (-3)\)  
6. \(-2 + 3\)

**Solve each equation.**

7. \(-12 + 8 = x\)
8. \(y = 4 + 5\)
9. \(z = 15 + (-5)\)

**Evaluate each expression if \( a = 2, b = -5, \) and \( c = -4.\)**

10. \(a + b\)
11. \(a + c\)
12. \(a + (-2)\)

**Money Matters** Krishana used a $5 bill to pay for a hamburger and soda that cost $2. She used the expression $5 + (-$2) to find out how much change she would get back. How much change did Krishana receive?

**Standardized Test Practice** What is the value of \( k + j \) if \( k = -7 \) and \( j = -5?\)

A \(-12\)  
B \(-2\)  
C \(12\)  
D \(2\)
Adding the additive inverse of an integer produces the same result as subtracting the integer.

<table>
<thead>
<tr>
<th>Subtracting Integers</th>
<th>To subtract an integer, add its additive inverse.</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 − 2 = 8 + (−2)</td>
<td>a − b = a + (−b)</td>
</tr>
</tbody>
</table>

**EXAMPLES**

A Find 6 − (−3).

Rewrite this subtraction problem as adding the inverse.
6 − (−3) = 6 + 3
6 − (−3) = 9

B Find −9 − 4.

Rewrite this subtraction problem as adding the inverse.
−9 − 4 = −9 + (−4)
−9 − 4 = −13

**Try These Together**

1. Find −4 − (−8).

HINT: Rewrite the subtraction as adding the inverse.

2. Solve g = 5 − (−2).

HINT: Rewrite as 5 plus the inverse of −2.

**PRACTICE**

Solve each equation.

3. h = −8 − 3

4. −12 − 6 = k

5. −6 − 4 = m

6. −2 − (−1) = n

7. 10 − (−3) = x

8. y = −15 − 7

Evaluate each expression if r = 2, s = −5, and t = −4.

9. r − 5

10. s − t

11. t − (−8)

12. 7 − r

13. r − s

14. 8 − s

15. r − (−2)

16. s − 6

17. 14 − t

18. **Sports** Oliver and his sister Sarah had a bicycle race. Sarah is younger and cannot ride as fast as Oliver. So Sarah started 2 meters in front of the starting line and Oliver started 5 meters behind the starting line. How many meters behind his sister did Oliver start?

19. **Standardized Test Practice** Find p if m − (−p) = 5 and m = 3.

A −4

B −5

C 3

D 2
When you multiply two integers, look to see if they have the same or different signs.

### Multiplying Integers with Different Signs
The product of two integers with different signs is negative.

### Multiplying Integers with the Same Sign
The product of two integers with the same sign is positive.

#### Examples

**A** Find \((-3) \times (-5)\).

These two integers have the same sign, so their product is positive.

\((-3) \times (-5) = 15\)

**B** Find the product of \(-7\) and \(3\).

These two integers have different signs, so their product is negative.

\((-7) \times 3 = -21\)

#### Try These Together

1. Solve \(g = 5(-2)\).

**HINT:** Are the signs of the factors the same or different?

2. Solve \(h = 4(-5)\).

**HINT:** What sign will the product have?

#### Practice

Solve each equation.

3. \(-10(-3)^2 = j\)

4. \(2(-6) = k\)

5. \(9(-5) = m\)

6. \(-4(-7) = n\)

7. \(-5(-8) = p\)

8. \(q = -12(-2)^2\)

9. \(r = -15(3^2)\)

10. \(t = 7(-3)\)

11. \(w = -8(-4)\)

Evaluate each expression if \(a = 3\), \(b = -8\), \(c = -2\), and \(d = 4\).

12. \(-3a\)

13. \(5c^2\)

14. \(-2bd\)

15. \(-15c\)

16. \(2ab\)

17. \(-2ad^2\)

18. **Game Shows**

Phil was a contestant on a game show where every time he answered incorrectly, he lost \$500. Phil answered incorrectly 3 times. Write an equation to show how much money Phil lost.

19. **Standardized Test Practice**

A golfer played 3 days in a row in a tournament and was 4 strokes under par each day. What integer represents the number of strokes under par this player was at the end of the tournament?

- **A** \(-12\)
- **B** \(12\)
- **C** \(-7\)
- **D** \(7\)
Dividing Integers

The rules for dividing integers are very similar to those for multiplying integers.

<table>
<thead>
<tr>
<th>Dividing Integers</th>
<th>The quotient of two integers with the same sign is positive.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>The quotient of two integers with different signs is negative.</td>
</tr>
</tbody>
</table>

The rules for dividing integers are very similar to those for multiplying integers.

### EXAMPLES

**A** Find the quotient of 15 and \(-5\).

*The signs of these two integers are different.*

*The quotient is negative.*

\[15 \div (-5) = -3\]

**B** Find \(-24 \div (-3)\).

*The signs of these two integers are the same.*

*The quotient is positive.*

\[-24 \div (-3) = 8\]

### Try These Together

1. Solve \(16 \div (-2) = a\).

*HINT: Are the signs the same or different?*

2. Solve \(b = \frac{-21}{-3}\).

*HINT: Compare the signs.*

### Practice

Solve each equation.

3. \(-8 \div (-4) = c\)

4. \(\frac{24}{-6} = d\)

5. \(\frac{-18}{-3} = e\)

6. \(f = -32 \div (-8)\)

7. \(g = -40 \div 8\)

8. \(h = -44 \div (-11)\)

9. \(\frac{54}{-9} = j\)

10. \(k = \frac{-9}{3}\)

11. \(m = -72 \div (-8)\)

12. \(-5 \div (-5) = n\)

### Evaluate each expression if \(a = 24\), \(b = -8\), and \(c = -4\).

13. \(a \div b\)

14. \(\frac{b}{c}\)

15. \(a \div (-12)\)

16. \(\frac{b}{2}\)

17. \(a \div (-1)\)

18. \(\frac{-32}{b}\)

19. **Entertainment** Tickets to the zoo for Larissa’s family cost $35. If there are 5 people in her family, what was the cost per person?

20. **Standardized Test Practice** What is the quotient of \(-45\) and \(9\)?

A 36  B -36  C 5  D -5

Coordinate Treasure Hunt

Starting at point $T$ on the coordinate plane below, follow the directions to find the location of a hidden treasure. Record your location at each point.

1. Add 1 to the $x$-coordinate and two to the $y$-coordinate. Where are you?

2. Divide both the $x$- and $y$-coordinates by $-2$.

3. Subtract 2 from the $x$-coordinate and $-5$ from the $y$-coordinate.

4. Add 6 to the $x$-coordinate and $-2$ to the $y$-coordinate.

5. Multiply the $x$-coordinate by 2 and the $y$-coordinate by $-5$.

What are the coordinates of the hidden treasure?

Answers are located on page 106.
Problems in the world outside the classroom usually are in the form of words. You translate these words into algebraic expressions.

<table>
<thead>
<tr>
<th>Writing Phrases as Expressions and Sentences as Equations</th>
<th>Addition:</th>
<th>Subtraction:</th>
<th>Multiplication:</th>
<th>Division:</th>
</tr>
</thead>
<tbody>
<tr>
<td>plus, sum, total, more than, increased by</td>
<td>minus, less, difference, less than, decreased by</td>
<td>times, product, multiplied, of, twice</td>
<td>divided, quotient</td>
<td></td>
</tr>
</tbody>
</table>

**Examples**

**A** Write this phrase as an algebraic expression: 3 less than twice \( p \).
You can write “twice \( p \)” as \( 2p \).
You can write “3 less than \( 2p \)” as \( 2p - 3 \).

**B** Write this sentence as an algebraic equation: The sum of \( t \) and the quotient of 2 and 8 is 3.25.
You can write the quotient of 2 and 8 as \( \frac{2}{8} \) or \( \frac{1}{4} \). The sum means to add \( t \) and \( \frac{2}{8} \).
Is suggests an equals sign.
\[
t + \frac{2}{8} = 3.25
\]

**Try These Together**

1. Write an algebraic expression for the sum of \( a \) and 9.
*HINT: What operation does the word sum suggest?*

2. Write an algebraic expression for the difference of \( x \) and 5.
*HINT: You can translate the words the difference of \( x \) and 5 as \( x \) minus 5.*

**Practice**

Write each phrase as an algebraic expression.

3. 8 more than \( w \)
4. \( g \) multiplied by 4
5. 18 less than \( y \)
6. the product of \( m \) and 7
7. twice \( z \)
8. 7 minus \( n \)

Write each sentence as an algebraic equation.

9. Five times a number is 15.
10. The sum of a number and three is 12.
11. Five more than 3 times a number is 29.
12. A number decreased by 8 is 11.

13. **Standardized Test Practice** Suppose an adult bottlenose dolphin weighs 800 pounds. This is 735 pounds more than a typical newborn bottlenose dolphin. Which equation could be used to find the weight of a typical newborn bottlenose dolphin?

**A** \( x - 735 = 800 \)  **B** \( x + 735 = 800 \)  **C** \( x - 800 = 735 \)  **D** \( x + 800 = 735 \)
Solving Addition and Subtraction Equations (pages 156–159)

To solve an equation and get the variable alone on one side, you can undo what has been done to the variable. The operations of addition and subtraction undo each other. To keep the two sides of an equation equal, you always do the same thing to each side.

<table>
<thead>
<tr>
<th>Subtraction Property of Equality</th>
<th>If you subtract the same number from each side of an equation, then the two sides remain equal.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( a = b )</td>
</tr>
<tr>
<td></td>
<td>( a - c = b - c )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Addition Property of Equality</th>
<th>If you add the same number to each side of an equation, then the two sides remain equal.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( a = b )</td>
</tr>
<tr>
<td></td>
<td>( a + c = b + c )</td>
</tr>
</tbody>
</table>

**EXAMPLES**

A  Solve \(7.3 = x + 4.2\).

The number 4.2 has been added to \(x\). To undo adding 4.2, you can subtract 4.2 from each side.

\[
7.3 = x + 4.2 \\
-4.2 = -4.2 \\
3.1 = x
\]

Check: Does 7.3 equal 3.1 + 4.2?

Yes.

B  Solve \(y - 5 = -8\).

The number 5 has been subtracted from \(y\). To undo subtracting 5, you can add 5 to each side.

\[
y - 5 = -8 \\
+5 = +5 \\
y = -3
\]

Check: Does \(-3 - 5\) equal \(-8\)?

Yes.

**Try These Together**

1. Solve \(s + 12 = 15\).

\(HINT: \) Subtract 12 from each side.

2. Solve \(t - 8 = 6\).

\(HINT: \) Add 8 to each side.

**PRACTICE**

**Solve each equation. Check your solution.**

3. \(34 = 24 + w\)  
4. \(8 + x = 9\)  
5. \(2 + y = 11\)  
6. \(z - 5 = 4\)  
7. \(-3 = r - 7\)  
8. \(25 = j - 30\)  
9. \(k + 18 = 26\)  
10. \(-8 + m = -4\)  
11. \(29 = 23 + n\)  
12. \(p + 4 = 17\)  
13. \(21 = f - 8\)  
14. \(8 = g + 16\)

15. A number decreased by 6 equals \(-4\). This means \(x - 6 = -4\). Solve this equation to find the number.

16. The sum of 3 and a number is \(-2\). Solve \(3 + y = -2\) to find the number.

17. **Standardized Test Practice**  Trejon went with her friends to a movie. She had \(\$10.00\) when she left. She came home with only \(\$4.50\). Solve the equation \(\$10.00 = \$4.50 + m\) to find out how much she spent.

\(A \$3.50 \quad B \$5.50 \quad C \$4.50 \quad D \$6.50\)
You can also solve an equation that has a number multiplied by a variable. In the equation $3x = 12$, $3x$ means 3 times the value of $x$. You undo multiplication by division.

<table>
<thead>
<tr>
<th>Division Property of Equality</th>
<th>If you divide each side of an equation by the same nonzero number, then the two sides remain equal.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$a = b$</td>
</tr>
<tr>
<td></td>
<td>$\frac{a}{c} = \frac{b}{c}$, $c \neq 0$</td>
</tr>
</tbody>
</table>

**EXAMPLES**

**A** Solve $2p = 6$.

To undo multiplying by 2, divide each side by 2.

$$\frac{2p}{2} = \frac{6}{2}$$  
$p = 3$

Check:

Does $2(3)$ equal 6?  
Yes.

**B** Solve $-5q = 45$.

To undo multiplying by $-5$, divide each side by $-5$.

$$\frac{-5q}{-5} = \frac{45}{-5}$$  
$q = -9$

Check:

Does $-5(-9)$ equal 45?  
Yes.

**Try These Together**

1. Solve $3x = 15$.

   **HINT:** Divide each side by the 3.

2. Solve $5t = 45$.

   **HINT:** Divide each side by 5.

**PRACTICE**

Solve each equation. Check your solution.

3. $10w = 50$  
4. $36 = 9z$  
5. $4s = 64$

6. $54 = 18m$  
7. $121 = -11n$  
8. $96 = 12k$

9. $81 = 9p$  
10. $100 = 5j$  
11. $4g = 20$

12. $-12 = 2h$  
13. $5d = -25$  
14. $14c = 56$

15. Eight times a number is $-56$. Find the solution of $8a = -56$.

16. When a number is multiplied by 30, the result is $-90$. Solve $30x = -90$ to find the number.

17. **Standardized Test Practice** Use the formula $4s = p$ to find the length $s$ of a side of a square with perimeter $p = 28$ cm.

   **A** 24 cm  
   **B** 6 cm  
   **C** 112 cm  
   **D** 7 cm

**Answers:** 1. 5  
2. 9  
3. 16  
4. 4  
5. 16  
6. 3  
7. 11  
8. 8  
9. 10  
10. 20  
11. 5  
12. 6  
13. 5  
14. 4  
15. 2  
16. 3  
17. 7  
18. 2  
19. 1  
20. 0

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*Mathematics: Applications and Concepts, Course 2*
Solving Two-Step Equations

**Examples**

A. Solve $3p + 5 = 23$.

*Undo the + 5 first by subtracting 5 from each side.*

$3p + 5 - 5 = 23 - 5$

$3p = 18$

Then undo the multiplying 3 by dividing by 3.

$3p = 18$

$p = 6$

Check by replacing $p$ by 6.

Does $3(6) + 5$ equal 23?

Yes.

B. Solve $-2q - 3 = -1$.

*Undo the - 3 first by adding 3 to each side.*

$-2q - 3 + 3 = -1 + 3$

$-2q = 2$

Undo the multiplying $-2$ by dividing by $-2$.

$\frac{-2q}{-2} = \frac{2}{-2}$

$q = -1$

Check by replacing $q$ by $-2$.

Does $-2(-1) - 3$ equal $-1$?

Yes.

**Try These Together**

1. Solve $2y + 1 = 5$.

*HINT: First subtract 1 from each side. Then divide each side by 2.*

2. Solve $8x - 12 = 12$.

*HINT: Add 12 to each side and then divide each side by 8.*

**Practice**

Solve each equation.

3. $4z + 18 = 34$

4. $9b - 60 = 30$

5. $3k + 11 = 35$

6. $-7m + 14 = 7$

7. $76 = 5d + 16$

8. $19 = 3e - 8$

9. $-6f - 2 = -20$

10. $100 = 11h - 10$

11. $98 = 9c + 17$

12. $-12 = -2r + 2$

13. $5t + 8 = 33$

14. $6s - 5 = 37$

15. Subtract seven from the product of a number and 3. The result is 29. Solve $3x - 7 = 29$ to find the number.

16. Multiply a number by 2 and add 6. The result is 18. Find the solution of $2y + 6 = 18$ to find the number.

17. **Internet** An Internet company charges $15 a month for Internet access. They also charge a one-time $20 setup fee. How many months would you have to use the company for the total cost to be $170? Solve the equation $15m + 20 = 170$, where $m$ is the number of months.

18. **Standardized Test Practice** Esteban saved his weekly allowance of $5 for a few weeks. He then spent $13 of it on comic books. If he has $7 left, how many weeks did he save his allowance? Solve the equation $5w - 13 = 7$, where $w$ is the number of weeks.

A. 2 weeks  B. 3 weeks  C. 4 weeks  D. 5 weeks

Answers: 1. 2  2. 3  3. 7  4. 0  5. 5  6. 1  7. 12  8. 0  9. 3  10. 9  11. 11  12. 15  13. 12  14. 11  15. 7  16. 18  17. 10  18. C
An inequality is a mathematical sentence that contains one or more of the symbols $<$, $>$, $\leq$, and $\geq$. You can solve an inequality much like you solve an equation. The solution of an inequality may include many numbers.

**EXAMPLE**

Solve the inequality $x + 4 \leq 6$. Graph the solution.

\[
x + 4 \leq 6 \quad \quad \quad \quad x + 4 - 4 \leq 6 - 4 \quad \quad \quad \quad x \leq 2
\]

The solution is $x \leq 2$, all numbers less than or equal to 2. To graph the solution, draw a number line. Since 2 is included in the solution, fill in a circle at 2. Then draw a thick arrow over the numbers to the left.

**Try These Together**

1. Solve $y - 2 < 8$ and graph the solution.  
   **HINT:** Add 2 to each side.

2. Solve $5x > 25$ and graph the solution.  
   **HINT:** Divide each side by 5.

**Practice**

Solve each inequality. Graph the solution on a number line.

3. $c + 4 < 7$  
4. $j - 5 \leq 3$  
5. $7 + r < 14$  
6. $g - 4 > 3$

7. $x + 5 > 4$  
8. $9z \geq 9$  
9. $4d \geq 24$  
10. $3f - 2 \leq 10$

Write an inequality for each sentence. Then solve the inequality.

11. Eight times a number is less than 24.
12. Eight less than a number is greater than or equal to 12.

13. **Sales** Ian is earning money for a school trip by selling frozen pizzas. Each pizza costs $8. Ian needs to earn at least $160 for the trip. Write an inequality for the number of pizzas Ian needs to sell to earn at least $160. Then solve the inequality.

14. **Standardized Test Practice** In order to be the president of the United States, you must be at least 35 years old. Which inequality shows the possible ages of presidents?

   A  $a \leq 35$  
   B  $a > 35$  
   C  $a < 35$  
   D  $a \geq 35$

A linear equation has a graph that is a straight line. A linear equation looks like \( y = ax + b \), where \( a \) and \( b \) are positive or negative numbers.

<table>
<thead>
<tr>
<th>Graphing a Linear Equation</th>
<th>To graph an equation:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>• Select a few values for ( x ). Substitute and solve for ( y ). Use each solution to write an ordered pair ((x, y)).</td>
</tr>
<tr>
<td></td>
<td>• Graph the points for two of the ((x, y)) pairs.</td>
</tr>
<tr>
<td></td>
<td>• Draw a line through the two points you graphed.</td>
</tr>
<tr>
<td></td>
<td>• Check by graphing the other ((x, y)) pairs to make sure that they are on the same line that you drew. Make corrections if they are not.</td>
</tr>
</tbody>
</table>

**Example**

For \( y = 2x \), find four ordered pairs that make the equation true.

Choose four values for \( x \), for example, 0, 1, 2, and 3.

Replace \( x \) and solve for \( y \).

\[
\begin{align*}
\text{Replace } x &= 0 \quad & y &= 2(0) = 0 \\
\text{result as an ordered pair: } (0, 0) \\
\text{Replace } x &= 1 \quad & y &= 2(1) = 2 \\
\text{result as an ordered pair: } (1, 2) \\
\text{Replace } x &= 2 \quad & y &= 2(2) = 4 \\
\text{result as an ordered pair: } (2, 4) \\
\text{Replace } x &= 3 \quad & y &= 2(3) = 6 \\
\text{result as an ordered pair: } (3, 6)
\end{align*}
\]

**Try These Together**

1. Graph \( y = 2x \).
2. Graph \( y = 3x - 5 \).

**HINT:** Use the four ordered pairs you found in the Example.

**HINT:** Choose four values for \( x \), such as 0, 1, 2, and 3. Then replace \( x \) with each value and solve for \( y \).

**Practice**

Graph each equation.

3. \( y = x - 2 \)  
4. \( y = 4x \)  
5. \( y = -4x + 8 \)  
6. \( y = 5x - 10 \)

Make a table of values for each sentence. Then write an equation.

Let \( x \) represent the first number and \( y \) represent the second number.

7. The second number is 2 less than the first.
8. The second number is 3 times the first.
9. The sum of the numbers is 5.
10. The second number is the product of 6 and the first number.

11. **Standardized Test Practice**

What is the equation for this graph?

- A \( y = 5x \)   
- B \( 2x - 1 = y \)   
- C \( y = 3x \)   
- D \( y = 4x - 2 \)
The change in \( y \) with respect to the change in \( x \) is called the **slope** of a line.

Slope is a number that tells how steep the line is.

\[
slope = \frac{\text{change in } y}{\text{change in } x}
\]

← vertical change, or rise

← horizontal change, or run

The slope is the same for any two points on a straight line. A line with a positive slope *rises* to the right. A line with a negative slope *falls* to the right.

### Example

Find the slope of the line.

\[
slope = \frac{\text{change in } y}{\text{change in } x}
\]

\[
= \frac{-1}{-1} \text{ or } 1
\]

*The slope of the line is 1.*

### Try These Together

Find the slope of each line.

1. [Graph]

2. A line that passes through the points \((0, 3)\) and \((7, 5)\)

*HINT: Find the change in \( y \) first, then find the change in \( x \).*

### Practice

Find the slope of the line that passes through each pair of points.

3. \((-2, 5), (6, -1)\)
4. \((0, 7), (9, 3)\)
5. \((-4, -1), (-8, -5)\)
6. \((2, -10), (4, 6)\)
7. \((9, 0), (3, -6)\)
8. \((5, -1), (8, 1)\)

9. **Standardized Test Practice** The slope of a line that passes through the points \((-2, 0)\) and \((x, 8)\) is 4. What is the value of \(x\)?

   A 4  B 0  C 2  D 4

*Answer: 1*
Function Map

Daniel has this map of Rocky Creek State Park. He is supposed to meet his friends at a campsite in the park. The campsite lies on the graph of the equation $y = -2x - 3$.

1. Which points on the map could possibly be the campsite?

2. If the campsite is in Quadrant II on the map, which point is it?

3. There is a scenic lookout that is also on the graph of the equation in Quadrant III. Which point is the scenic lookout?

4. The entrance to the park is on the graph of the function in Quadrant IV. Which point is the entrance to the park?

Answers are located on page 107.
A prime number is a whole number greater than 1 that has exactly two factors, 1 and itself. A composite number is a whole number greater than 1 that has more than two factors. Every composite number can be written as the product of prime numbers in exactly one way if you ignore the order of the factors. This product is called the prime factorization of the number.

### Example

#### Finding Prime Factorization

<table>
<thead>
<tr>
<th>Finding Prime Factorization</th>
<th>Method 1: Use a factor tree.</th>
<th>Method 2: Divide by prime numbers until the quotient is prime. Use a calculator, if necessary.</th>
</tr>
</thead>
<tbody>
<tr>
<td>36</td>
<td><img src="factor_tree.png" alt="Factor Tree" /></td>
<td>$36 \div 2 = 18; 18 \div 2 = 9; 9 \div 3 = 3$</td>
</tr>
</tbody>
</table>

The prime factorization of 36 is $2 \times 2 \times 3 \times 3$.

### Try These Together

1. Is 23 composite or prime?  
   *Hint: Test for divisibility by 2, 3, 5, 7, and 11.*

2. Use a factor tree to find the prime factorization of 28.  
   *Hint: You can divide by 2 and then by 2 again.*

### Practice

#### Determine whether each number is composite or prime.

3. 51  
4. 228  
5. 227  
6. 73  
7. 154

#### Use a factor tree to find the prime factorization of each number.

8. 64  
9. 93  
10. 54  
11. 125  
12. 244

#### Use your calculator to find the prime factors of each number. Then write the prime factorization of each number.

13. 84  
14. 96  
15. 150  
16. 30  
17. 232  
18. 245

19. Maps Rhode Island is the smallest state in the United States. It only covers 3,188 square kilometers. Find the prime factorization of 3,188.

20. Standardized Test Practice Which number is a factor of both 21 and 36?  
   **A** 4  
   **B** 3  
   **C** 9  
   **D** 12

   *Answers: A prime number, B prime number, C composite, D prime number*
The greatest common factor (GCF) of two or more numbers is the greatest number that is a factor of each number.

### Finding the Greatest Common Factor

To find the GCF of two or more numbers:
- **Method 1:** List the factors of each number and then identify the common factors. The greatest of these common factors is the GCF.
- **Method 2:** Write the prime factorization of each number, or divide by prime numbers until the quotient is prime. Then identify all common prime factors and find their product, which is the GCF.

### Examples

**A** Find the GCF of 12, 20, and 36 by listing factors.
- **factors of 12:** 1, 2, 3, 4, 6, 12
- **factors of 20:** 1, 2, 4, 5, 10, 20
- **factors of 36:** 1, 2, 3, 4, 6, 9, 12, 18, 36

The greatest of the common factors is 4, which is the GCF of 12, 20, and 36.

**B** Find the GCF of 27 and 90 by using prime factorization.
- **Prime factorization of 27:** $3 \times 3 \times 3$
- **Prime factorization of 90:** $2 \times 3 \times 3 \times 5$

The common prime factors are 3 and 3. Their product is 9. The GCF of 27 and 90 is 9.

### Try These Together

1. Find the GCF of 12 and 16 by listing factors. **HINT:** Circle the factors common to 12 and 16. Then choose the greatest of those circled.
2. Find the GCF of $15 = 3 \times 5$ and $50 = 2 \times 5^2$ by listing common prime factors. **HINT:** There is only one common prime factor.

### Practice

**Find the GCF of each set of numbers by listing factors.**
3. 54, 81  
4. 72, 90  
5. 132, 144  
6. 20, 36, 44

**Find the GCF of each set of numbers by listing common prime factors.**
7. $9 = 3^2$  
$36 = 2^2 \times 3^2$  
8. $45 = 3^2 \times 5$  
$81 = 3^4$

**Find the GCF of each set of numbers by writing prime factorizations.**
9. 12, 48  
10. 36, 54  
11. 60, 42

12. **Life Science** The smallest adult male gorillas weigh about 135 kilograms. The smallest adult female gorillas weigh about 70 kilograms. What is the greatest common factor of these two numbers?

13. **Standardized Test Practice** Find the greatest common factor of 96 and 360.
   - A 5  
   - B 12  
   - C 36  
   - D 24

*Answers: 1. 4  2. 5  3. 27  4. 18  5. 4  6. 9  7. 9  8. 9  9. 12  10. 18  11. 6  12. 18  13. 3  14. 24*
You can simplify the fraction $\frac{3}{9}$ by dividing both the numerator and denominator by 3. A fraction is in **simplest form** when the GCF of the numerator and denominator is 1.

<table>
<thead>
<tr>
<th>Simplifying Fractions</th>
<th>To write a fraction in simplest form:</th>
</tr>
</thead>
<tbody>
<tr>
<td>• find the GCF of the numerator and denominator,</td>
<td></td>
</tr>
<tr>
<td>• divide both the numerator and denominator by the GCF, and</td>
<td></td>
</tr>
<tr>
<td>• write the resulting fraction.</td>
<td></td>
</tr>
</tbody>
</table>

### EXAMPLES

**A** Express $\frac{6}{12}$ in simplest form.

*The GCF of 6 and 12 is 6.*

*Divide the numerator (6) by 6 to get 1.*

*Divide the denominator (12) by 6 to get 2.*

$\frac{6}{12}$ in simplest form is $\frac{1}{2}$.

**B** Express $\frac{18}{24}$ in simplest form.

*The GCF of 18 and 24 is 6.*

*Divide 18 by 6. Divide 24 by 6.*

$\frac{18}{24}$ in simplest form is $\frac{3}{4}$.

### Try These Together

1. Express $\frac{25}{45}$ in simplest form.  
   *HINT: The GCF of 25 and 45 is 5.*

2. Express $\frac{3}{15}$ in simplest form.  
   *HINT: What is the GCF of 3 and 15?*

### PRACTICE

Express each fraction in simplest form.

3.  $\frac{82}{94}$  
4.  $\frac{54}{63}$  
5.  $\frac{48}{16}$  
6.  $\frac{55}{105}$

7.  $\frac{12}{60}$  
8.  $\frac{10}{148}$  
9.  $\frac{14}{62}$  
10.  $\frac{8}{72}$

11. **Life Science** There are 2,900 species of jellyfish. They are made up of 2 classes, the hydrozoan and the scyphozoan. There are 200 species of scyphozoan. Express the number of species of scyphozoan as a fraction of all jellyfish species in simplest form.

12. **Standardized Test Practice** Akikta has $1,200 in his checking account and $300 in his savings account. Express the amount of money in his savings account as a fraction of the amount of money in his checking account in simplest form.

   **A**  $\frac{1}{4}$  
   **B**  $\frac{3}{4}$  
   **C**  $\frac{1}{2}$  
   **D**  $\frac{1}{3}$
Decimals and Fractions (pages 210–213)

Any fraction can be written as a decimal by using division.

<table>
<thead>
<tr>
<th>Write a Fraction as a Decimal</th>
<th>Use paper and pencil to write $\frac{4}{5}$ as a decimal. $\frac{4}{5}$ means $4 \div 5$. Divide 4 by 5, and the quotient is the decimal you want to find, 0.8.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Repeating Decimals</td>
<td>Decimals like 0.333333 . . . are called <strong>repeating decimals</strong> because the digits repeat. <strong>Bar notation</strong> can be used to indicate that decimals repeat. 0.6666666 . . . = $\frac{2}{3}$, 0.277777 . . . = $\frac{9}{33}$, 0.737373 . . . = $\frac{7}{9}$. Bar notation is useful because some fractions, when written as decimals, are repeating decimals. For example, $\frac{2}{3} = 0.\overline{6}$.</td>
</tr>
</tbody>
</table>

**EXAMPLES**

Express the fractions as decimals. Use bar notation for repeating decimals.

<table>
<thead>
<tr>
<th>A</th>
<th>$\frac{3}{5}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{3}{5} = 3 \div 5$</td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td></td>
</tr>
<tr>
<td>53.0</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td></td>
</tr>
<tr>
<td>0 Therefore, $\frac{3}{5} = 0.6$.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B</th>
<th>$\frac{3}{11}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{3}{11} = 3 \div 11$</td>
<td></td>
</tr>
<tr>
<td>0.2727 ...</td>
<td></td>
</tr>
<tr>
<td>11)3.00</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>80</td>
<td></td>
</tr>
<tr>
<td>77</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td></td>
</tr>
<tr>
<td>8 Therefore, $\frac{3}{11} = 0.\overline{27}$.</td>
<td></td>
</tr>
</tbody>
</table>

Try These Together

**Express each fraction or mixed number as a decimal. If the decimal is a repeating decimal, use bar notation.**

1. $\frac{1}{6}$

**HINT:** Divide 1 by 6.

2. $\frac{4}{7}$

**HINT:** The whole number is written to the left of the decimal point.

3. $\frac{3}{6}$

4. $\frac{2}{9}$

5. $\frac{12}{25}$

6. $5 \frac{2}{3}$

7. $\frac{8}{9}$

8. $7 \frac{1}{4}$

9. **Standardized Test Practice** Suppose that $\frac{1}{8}$ of D’andre’s class scored As on their science exam. Express this fraction as a decimal.

**A** 0.215

**B** 0.125

**C** 0.252

**D** 0.115
A percent is a ratio that compares a number to 100. Fractions and percents are ratios that represent the same number.

Expressing a Ratio as a Percent

\[
\frac{n}{100} = n\% 
\]
To express a ratio as a percent, first write the ratio as a fraction with a denominator of 100. Then rewrite \( \frac{n}{100} \) as \( n\% \).

Expressing a Fraction as a Percent

To express a fraction as a percent, multiply both numerator and denominator by the same factor to rewrite the fraction as an equivalent fraction with a denominator of 100.

**EXAMPLES**

A  Express as a percent: 37 students out of 100.

Write the ratio as a fraction: \( \frac{37}{100} \).

\( \frac{37}{100} \) is 37%.

B  Express \( \frac{7}{25} \) as a percent.

To rewrite \( \frac{7}{25} \) as an equivalent fraction with a denominator of 100, multiply both numerator and denominator by 4, since 100 ÷ 25 = 4.

\[
\frac{7 \times 4}{25 \times 4} = \frac{28}{100}, \quad \frac{28}{100} \text{ is } 28\%.
\]

\[
\frac{7}{25} = 28\%
\]

**Try These Together**

1. Express as a percent: 32.5 square miles in 100.

   **HINT:** Write as a fraction with a denominator of 100.

2. Express \( \frac{3}{5} \) as a percent.

   **HINT:** Recall that \( \frac{n}{100} = n\% \).

**PRACTICE**

Express each ratio or fraction as a percent.

3. 62 out of 100

4. \( \frac{1}{4} \)

5. \( \frac{12}{100} \)

6. \( \frac{2}{5} \)

7. $55 per $100

8. \( \frac{13}{20} \)

9. **Computers** 78 out of 100 computers at Tina’s school have CD-ROM drives. Express 78 out of 100 as a percent.

10. **Standardized Test Practice** In Enrique’s school, 61 out of every 100 students eat a hot lunch. Express this ratio as a percent.

   **A** 3.9%
   **B** 6.1%
   **C** 61%
   **D** 39%
Any decimal can also be written as a fraction. You can use this to express any decimal as a percent.

### Writing a Decimal as a Percent
To write 0.32 as a percent, multiply the decimal by 100 and add the percent symbol. So, \(0.32 = \frac{0.32}{1} \times 100 = 32\%\).

### Writing a Percent as a Decimal
To write a percent as a decimal, divide the percent by 100 and remove the percent symbol. 64\% = \frac{64}{100} = 0.64.

### EXAMPLES

**A** Write 0.72 as a percent.

\[
0.72 = 72\%
\]

**B** Write 57\% as a decimal.

\[
57\% = \frac{57}{100} = 0.57
\]

### Try These Together

**Express each decimal as a percent.**

1. 0.25
2. 0.76

**Express each percent as a decimal.**

3. 0.54
4. 0.67
5. 0.1
6. 0.08
7. 0.42
8. 0.17
9. 48\%
10. 75\%
11. 9\%
12. 23\%
13. 35\%
14. 99.8\%
15. 4\%
16. 15.1\%

### 17. What decimal is equivalent to 39.5\%?

### 18. Write the percent that is equivalent to 0.652.

### 19. Recycling

In a recent year, the aluminum recycling rate was 62.8\%.

Write this percent as a decimal.

### 20. Standardized Test Practice

In the mid-1990s, 48\% of the working people in the world were employed in agriculture. How is this percent written as a decimal?

**A** 0.048
**B** 48
**C** 4.8
**D** 0.48
When you multiply a number by the whole numbers 0, 1, 2, 3, 4, and so on, you get multiples of the number. The least common multiple (LCM) of two or more numbers is the least of their common multiples, other than zero.

<table>
<thead>
<tr>
<th>Finding the Least Common Multiple (LCM)</th>
<th>To find the least common multiple of two or more numbers,</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>• make a list of several multiples of each number. Then identify the</td>
</tr>
<tr>
<td></td>
<td>common multiples. The least of these is the LCM.</td>
</tr>
<tr>
<td></td>
<td>OR</td>
</tr>
<tr>
<td></td>
<td>• write the prime factorization of each number. Write each prime factor</td>
</tr>
<tr>
<td></td>
<td>as a multiplier the greatest number of times it appears in any one of</td>
</tr>
<tr>
<td></td>
<td>the numbers.</td>
</tr>
<tr>
<td></td>
<td>OR</td>
</tr>
<tr>
<td></td>
<td>• divide by prime factors until the quotients are prime. Then multiply</td>
</tr>
<tr>
<td></td>
<td>the divisors and prime quotients to get the LCM.</td>
</tr>
</tbody>
</table>

Find the LCM of 6, 36, and 40 by writing prime factorizations.

\[6 = 2 \times 3 \quad 36 = 2 \times 2 \times 3 \times 3 \quad 40 = 2 \times 2 \times 2 \times 5\]

Write each prime factor (2, 3, 5) as a multiplier the greatest number of times it appears in any one number. The factor 2 appears three times in 40. The factor 3 appears twice in 36. The factor 5 appears once in 40. The product of \(2 \times 2 \times 2 \times 3 \times 3 \times 5\), or 360, is the least common multiple of 6, 36, and 40.

### Try These Together

1. Find the LCM of 12 and 30 by listing multiples.
   *HINT: Look for the least common multiple in the two lists.*

2. Find the LCM of 12 and 14 by writing prime factorizations.
   *HINT: Remember to write each prime factor as a multiplier the greatest number of times it appears in any one of the numbers.*

### Practice

**Find the LCM of each set of numbers by listing multiples.**

3. 3, 10  
4. 6, 8  
5. 9, 12  
6. 3, 5, 6  
7. 4, 5, 10  
8. 5, 15

**Find the LCM of each set of numbers by writing prime factorizations or dividing by prime numbers.**

9. 6, 9  
10. 12, 18  
11. 8, 14  
12. 10, 36  
13. 20, 96  
14. 4, 6, 15

15. **Entertainment** Every 10 years, the people of Oberammergau, Germany, put on a special play. Rhonda’s family travels to Germany every 3 years. If Rhonda’s family was in Germany in the year 2000 and the play was on, what is the next year that the play will be on when Rhonda’s family is in Germany?

16. **Standardized Test Practice** What is the least common multiple of 50 and 60?

   - A 200
   - B 400
   - C 300
   - D 500

---

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To compare fractions, rewrite each fraction using the same denominator. Then you only need to compare the numerators.

### Finding the Least Common Denominator

A common denominator is a common multiple of the denominators of two or more fractions. The least common denominator (LCD) is the least common multiple (LCM) of the denominators of two or more fractions.

To compare two fractions:
- find the LCM of the denominators, then
- rewrite each fraction using this LCM as the LCD. Compare the numerators.

#### Examples

**A** Find the LCD for \( \frac{5}{6} \) and \( \frac{9}{10} \).

*The LCM of 6 and 10 is 30, so the LCD for \( \frac{5}{6} \) and \( \frac{9}{10} \) is 30.*

**B** Is \( \frac{5}{6} <, >, \) or = \( \frac{9}{10} \)?

*Rewrite each fraction with the LCD of 30.*

\[
\frac{5}{6} = \frac{25}{30}, \quad \frac{9}{10} = \frac{27}{30}
\]

*Since \( \frac{25}{30} < \frac{27}{30} \), \( \frac{5}{6} < \frac{9}{10} \).*

#### Try These Together

1. Find the LCD for \( \frac{3}{4} \) and \( \frac{2}{3} \).

*HINT: Find the LCM of 4 and 3.*

2. Is \( \frac{5}{8} \) <, >, or = 0.4?

*HINT: Write both rational numbers as fractions with the same denominator or as decimals.*

#### Practice

**Find the LCD for each pair of fractions.**

3. \( \frac{5}{12}, \frac{3}{8} \)
4. \( \frac{2}{5}, \frac{4}{7} \)
5. \( \frac{4}{15}, \frac{1}{3} \)
6. \( \frac{1}{6}, \frac{1}{9} \)
7. \( \frac{1}{6}, \frac{5}{7} \)
8. \( \frac{19}{30}, \frac{7}{10} \)
9. \( \frac{9}{16}, \frac{1}{4} \)
10. \( \frac{5}{36}, \frac{11}{24} \)

**Replace each \( \bullet \) with <, >, or = to make a true sentence.**

11. \( \frac{8}{9} \bullet \frac{5}{6} \)
12. \( \frac{2}{3} \bullet \frac{8}{13} \)
13. \( \frac{5}{6} \bullet 0.75 \)
14. \( \frac{3}{5} \bullet \frac{5}{8} \)
15. \( \frac{2}{7} \bullet 0.25 \)
16. \( 0.7 \bullet \frac{14}{20} \)
17. \( \frac{5}{11} \bullet \frac{13}{22} \)
18. \( \frac{15}{48} \bullet \frac{3}{8} \)

19. **Standardized Test Practice**

What is the least common denominator for \( \frac{1}{8} \) and \( \frac{5}{6} \)?

A 36 \hspace{1cm} B 24 \hspace{1cm} C 18 \hspace{1cm} D 45

**Answers:** 12, 24.
Chapter 5 Review

Fraction Ladder

Build a ladder out of the following list of fractions. Place the fractions in order from least to greatest on the ladder from bottom to top.

\[
\begin{align*}
\frac{1}{2} \\
\frac{19}{20} \\
\frac{4}{5} \\
\frac{7}{8} \\
\frac{3}{4} \\
\frac{1}{12} \\
\frac{1}{3} \\
\frac{3}{10}
\end{align*}
\]

Answers are located on page 107.
Estimating helps you find answers when you only need an approximate solution. Estimating before you do exact calculations helps you check your work.

<table>
<thead>
<tr>
<th>Estimating with Fractions and Mixed Numbers</th>
<th>To estimate the sum or difference of fractions:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>• Round each fraction to 0, 1/2, or 1, whichever is closest.</td>
</tr>
<tr>
<td></td>
<td>• Calculate with your rounded fractions.</td>
</tr>
</tbody>
</table>

To estimate the sum, difference, or product of mixed numbers:

- Round each mixed number to the nearest whole number.
- Calculate with these whole numbers.

### Examples

**A**  Estimate \( \frac{4}{7} - \frac{1}{5} \).

Think: Half of 7 is 3 1/2, so \( \frac{4}{7} \) is close to \( \frac{1}{2} \).

\( \frac{1}{5} \) is close to 0.

Calculate with the rounded fractions: \( \frac{1}{2} - 0 = \frac{1}{2} \).

\( \frac{4}{7} - \frac{1}{5} \) is about \( \frac{1}{2} \).

**B**  Estimate \( 2\frac{7}{8} + 1\frac{1}{6} \).

Think: \( 2\frac{7}{8} \) is close to 3.

\( 1\frac{1}{6} \) is close to 1.

Calculate with the rounded numbers: \( 3 + 1 = 4. \)

\( 2\frac{7}{8} + 1\frac{1}{6} \) is about 4.

### Try These Together

1. Round \( 2\frac{2}{9} \) to 0, \( \frac{1}{2} \), or 1.

*HINT: \( 4\frac{1}{2} \) ninths equals \( \frac{1}{2} \).*

2. Round \( 1\frac{11}{12} \) to 0, \( \frac{1}{2} \), or 1.

*HINT: \( \frac{6}{12} \) is equal to \( \frac{1}{2} \).*

### Practice

Round each fraction to 0, \( \frac{1}{2} \), or 1.

3. \( \frac{1}{8} \)  
4. \( \frac{9}{16} \)  
5. \( \frac{8}{9} \)  
6. \( \frac{3}{7} \)

Round to the nearest whole number.

7. \( 2\frac{3}{4} \)  
8. \( 5\frac{1}{6} \)  
9. \( 4\frac{2}{5} \)  
10. \( 8\frac{7}{8} \)

Estimate.

11. \( \frac{5}{8} + \frac{1}{6} \)  
12. \( 1\frac{10}{11} + \frac{9}{10} \)  
13. \( 4\frac{1}{5} - 3\frac{2}{9} \)  
14. \( \frac{1}{11} \times \frac{4}{5} \)

15. **Standardized Test Practice**  Estimate the difference between \( 5\frac{4}{5} \) and \( 2\frac{2}{7} \).

   **A** 3  
   **B** 4  
   **C** 6  
   **D** 5

**Answers:** 1. 0 2. 1 3. 2 4. 3 5. 4 6. 5 7. 6 8. 7. 9. 8 10. 9 11. 10 12. 11 13. 12 14. 13 15. 14
To add or subtract fractions, the denominators must be the same.

### Examples

**A** Add $\frac{3}{4} + \frac{5}{6}$.

Remember that the least common denominator of 4 and 6 is their least common multiple (12).

Multiply to rename with the LCD:

$$\frac{3 \times 3}{4 \times 3} + \frac{5 \times 2}{6 \times 2}.$$  

Add $\frac{9}{12} + \frac{10}{12} = \frac{19}{12}$ and simplify: $1\frac{7}{12}$.

**B** Subtract $\frac{2}{3} - \frac{1}{6}$.

The LCD is 6.

$$\frac{2}{3} - \frac{1}{6} = \frac{2 \times 2}{3 \times 2} - \frac{1}{6}$$  

Multiply.

$$= \frac{4}{6} - \frac{1}{6}$$  

= $\frac{3}{6}$  

Subtract.

$$= \frac{1}{2}$$  

Simplify.

### Try These Together

1. Add $\frac{1}{8} + \frac{5}{8}$.

*HINT:* These have a common denominator already.

2. Subtract $\frac{5}{16} - \frac{1}{4}$.

*HINT:* The LCD is 16.

### Practice

**Add or subtract. Write each sum or difference in simplest form.**

3. $\frac{5}{7} - \frac{2}{3}$  
4. $\frac{1}{6} + \frac{3}{4}$  
5. $\frac{7}{18} + \frac{2}{9}$  
6. $\frac{15}{27} - \frac{1}{3}$

**Solve each equation. Write the solution in simplest form.**

7. $\frac{1}{7} + \frac{2}{3} = y$  
8. $b = \frac{1}{4} + \frac{7}{10}$  
9. $\frac{6}{15} + \frac{1}{2} = g$  
10. $h = \frac{5}{9} - \frac{1}{4}$

11. **Cooking** A recipe calls for $\frac{1}{2}$ pound of chocolate chips and $\frac{1}{4}$ pound of butterscotch chips. How many pounds of chips does it call for all together?

12. **Standardized Test Practice** Mr. Jensen is a flight attendant. $\frac{1}{4}$ of his uniforms are black and $\frac{5}{14}$ of his uniforms are red. What fraction of his uniforms are black or red?

**A** $\frac{9}{20}$  
**B** $\frac{2}{3}$  
**C** $\frac{6}{18}$  
**D** $\frac{17}{28}$

*Answers: 1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12.*
You can add or subtract mixed numbers with steps similar to those you used for adding or subtracting fractions.

### Adding and Subtracting Mixed Numbers

To add or subtract mixed numbers:

- Add or subtract the fraction parts, renaming them if necessary.
- Add or subtract the whole numbers and simplify.

#### EXAMPLE

Subtract $3\frac{1}{4} - 2\frac{5}{8}$.

$3\frac{1}{4} - 2\frac{5}{8} = 3\frac{2}{8} - 2\frac{5}{8}$  

*The LCD is 8.*

$= \frac{26}{8} + \frac{2}{8} - \frac{25}{8}$  

*Rename, since you cannot subtract $\frac{5}{8}$ from $\frac{2}{8}$.*

$= \frac{210}{8} - \frac{25}{8}$  

$= \frac{5}{8}$  

*Subtract.*

$\frac{10}{8} - \frac{5}{8} = \frac{5}{8}$, $2 - 2 = 0$

#### Try These Together

1. Complete: $2\frac{1}{5} = 1\frac{\_}{5}$.
   
   *HINT: $2\frac{1}{5}$ is $1 + 1 + \frac{1}{5}$. How many fifths is 1?*

2. Complete: $4\frac{5}{8} = 3\frac{\_}{8}$.
   
   *HINT: How many eighths is 1?*

#### Complete.

3. $8\frac{1}{6} = 7\frac{\_}{6}$  

4. $6\frac{10}{7} = 7\frac{\_}{7}$  

5. $5\frac{2}{3} = 4\frac{\_}{3}$  

6. $8\frac{7}{5} = 2\frac{\_}{5}$

#### Add or subtract. Write each sum or difference in simplest form.

7. $3\frac{5}{8} + 1\frac{1}{8}$  

8. $7\frac{3}{5} - 5\frac{2}{5}$  

9. $2\frac{4}{9} + 4\frac{1}{3}$  

10. $5\frac{1}{4} - 3\frac{2}{7}$

11. **Standardized Test Practice**

   Dierdre cut off and discarded $3\frac{1}{8}$ inches from a $12\frac{1}{2}$-inch-long piece of wrapping paper to wrap a gift. How long was the piece of wrapping paper she used?

   **A** 16\frac{3}{8} inches  
   **B** 9\frac{3}{8} inches  
   **C** 15\frac{5}{8} inches  
   **D** 10\frac{1}{8} inches

---

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You can multiply fractions and mixed numbers that have the same or different denominators.

### Multiplying Fractions

To multiply fractions, multiply the numerators and then multiply the denominators. 
\[
\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd} \quad (b \text{ and } d \text{ are not equal to } 0.)
\]

### Multiplying Mixed Numbers

To multiply mixed numbers, rename each mixed number as an improper fraction. Then multiply the fractions.

### Examples

**A**
\[
\frac{3}{4} \times \frac{2}{9}
\]
\[
\frac{3}{4} \times \frac{2}{9} = \frac{3 \times 2}{4 \times 9} = \frac{6}{36} = \frac{1}{6}
\]

**B**
\[
1\frac{1}{4} \times 3\frac{2}{5}
\]
\[
1\frac{1}{4} \times 3\frac{2}{5} = \frac{5}{4} \times \frac{17}{5} = \frac{35}{2} = 17\frac{1}{2}
\]

### Try These Together

1. \(\frac{1}{5} \times \frac{3}{5}\)

   **HINT:** Multiply numerators and then multiply denominators.

2. \(\frac{5}{8} \times \frac{4}{5}\)

   **HINT:** Divide numerators and denominators by any GCFs to simplify before you multiply.

### Practice

**Multiply. Write each product in simplest form.**

3. \(\frac{3}{7} \times \frac{1}{3}\)

4. \(\frac{5}{9} \times \frac{1}{10}\)

5. \(\frac{2}{11} \times \frac{1}{5}\)

6. \(2\frac{1}{2} \times 1\frac{3}{4}\)

7. \(\frac{9}{10} \times \frac{6}{7}\)

8. \(\frac{6}{13} \times \frac{1}{2}\)

9. \(\frac{5}{8} \times \frac{2}{5}\)

10. \(3\frac{2}{3} \times 2\frac{1}{3}\)

**Solve each equation. Write the solution in simplest form.**

11. \(1\frac{2}{5} \times 2 = r\)

12. \(2\frac{1}{4} \times \frac{4}{9} = s\)

13. \(4\frac{1}{2} \times \frac{1}{12} = t\)

14. **Standardized Test Practice**

   For a science experiment, Evan needs 3 pieces of string that are each 5\(\frac{1}{2}\) inches long. How many inches of string does he need total?

   - **A** 66\(\frac{1}{4}\)
   - **B** 33\(\frac{1}{4}\)
   - **C** 15\(\frac{1}{2}\)
   - **D** 16\(\frac{1}{2}\)

   **Answers:** 1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. 14. 15. 16. 17. 18. 19. 20. 21. 22. 23. 24. 25. 26. 27. 28. 29. 30. 31. 32. 33. 34. 35. 36. 37. 38. 39. 40. 41. 42. 43. 44. 45. 46. 47. 48. 49. 50. 51. 52. 53. 54. 55. 56. 57. 58. 59. 60. 61. 62. 63. 64. 65. 66. 67. 68. 69. 70. 71. 72. 73. 74. 75. 76. 77. 78. 79. 80. 81. 82. 83. 84. 85. 86. 87. 88. 89. 90. 91. 92. 93. 94. 95. 96. 97. 98. 99. 100.
Addition and multiplication of fractions have the same properties as addition and multiplication of whole numbers. Two numbers whose product is 1 are multiplicative inverses, or reciprocals.

| Multiplicative Inverse Property | For all fractions $\frac{a}{b}$, where $a, b \neq 0$, $\frac{a}{b} \times \frac{b}{a} = 1$. |
| Multiplication Property of Equality | If $a = b$, then $ac = bc$. |

**EXAMPLES**

A Find the multiplicative inverse of $\frac{2}{3}$.

$\frac{2}{3} \times \frac{3}{2} = 1$ What number can you multiply by $\frac{2}{3}$ to get 1?

Since $\frac{2}{3} \times \frac{3}{2} = 1$, the multiplicative inverse of $\frac{2}{3}$ is $\frac{3}{2}$.

B Solve $\frac{q}{7} = 6$.

You can undo dividing by 7 by multiplying each side of the equation by 7.

$\frac{q}{7} \times 7 = 6 \times 7$

$q = 42$

**Try These Together**

Solve each equation.

1. $7 = \frac{y}{2}$

HINT: Multiply to undo dividing.

2. $\frac{3}{4}x = -6$

HINT: What is the multiplicative inverse of $\frac{3}{4}$?

**PRACTICE**

Solve each equation. Write the solution in simplest form.

3. $\frac{b}{6} = 2$

4. $-2 = \frac{3}{5}t$

5. $\frac{d}{3} = 9$

6. $\frac{1}{8}z = \frac{2}{7}$

7. **Standardized Test Practice** What is the reciprocal of $4\frac{1}{3}$?

A $\frac{3}{1}$

B $\frac{13}{3}$

C $\frac{1}{12}$

D $\frac{3}{13}$

**Answers:** 1. 14 2. 8 3. $\frac{2}{3}$ 4. $\frac{3}{8}$ 5. $\frac{5}{2}$ 6. $\frac{3}{7}$ 7. A
To divide by a fraction, multiply by its multiplicative inverse or reciprocal.

To divide by a fraction, multiply by its multiplicative inverse or reciprocal.

| Dividing by a Fraction | You can rewrite \( \frac{a}{b} \div \frac{c}{d} \) as \( \frac{a}{b} \times \frac{d}{c} \), where \( b, c, \) and \( d \neq 0 \). |

**EXAMPLES**

**A** Find \( \frac{2}{7} \div \frac{3}{5} \).

\[
\frac{2}{7} \div \frac{3}{5} = \frac{2}{7} \times \frac{5}{3} = \frac{10}{21}
\]

\( \frac{5}{3} \) is the multiplicative inverse of \( \frac{3}{5} \).

**B** Find \( 3\frac{1}{2} \div 5\frac{4}{9} \).

\[
3\frac{1}{2} \div 5\frac{4}{9} = \frac{7}{2} \div \frac{49}{9} = \frac{2}{7} \times \frac{9}{49} = \frac{9}{14}
\]

Rewrite the improper fractions as mixed numbers.

The GCF of 7 and 49 is 7.

Multiply.

**Try These Together**

1. Find \( \frac{3}{8} \div 3 \).

\( HINT: Rewrite \div 3 as \times \frac{1}{3} \).

2. Find \( \frac{2}{5} \div \frac{2}{3} \).

\( HINT: Rewrite \div \frac{2}{3} as \times \frac{3}{2} \).

**PRACTICE**

Divide. Write each quotient in simplest form.

3. \( \frac{5}{7} \div \frac{4}{7} \)  
4. \( \frac{8}{11} \div \frac{3}{4} \)  
5. \( \frac{4}{5} \div 2\frac{2}{3} \)  
6. \( 4\frac{4}{7} \div \frac{4}{5} \)

Solve each equation.

7. \( r = \frac{4}{7} \div 2 \)  
8. \( \frac{8}{9} \div 3\frac{1}{4} = s \)  
9. \( t = \frac{5}{6} \div \frac{2}{3} \)  
10. \( w = \frac{1}{3} \div \frac{1}{2} \)

11. Standardized Test Practice Taina has \( 3\frac{1}{5} \) yards of material that she wants to split into 4 pieces of equal length for a project. How long will each piece be?

A \( \frac{7}{9} \) yd  
B \( 1\frac{2}{5} \) yd  
C \( 3\frac{1}{36} \) yd  
D \( \frac{3}{4} \) yd

**Answers:** 1. \( \frac{11}{5} \), 2. \( \frac{9}{10} \), 3. \( \frac{7}{11} \), 4. \( \frac{2}{5} \), 5. \( \frac{6}{9} \), 6. \( \frac{1}{6} \), 7. \( \frac{5}{1} \), 8. \( \frac{1}{2} \), 9. \( \frac{1}{3} \), 10. \( \frac{1}{2} \), 11. \( \frac{7}{9} \), 12. \( \frac{2}{5} \), 13. \( \frac{3}{4} \), 14. \( \frac{5}{6} \), 15. \( \frac{1}{3} \), 16. \( \frac{1}{2} \), 17. \( \frac{1}{3} \), 18. \( \frac{2}{3} \), 19. \( \frac{7}{9} \), 20. \( \frac{2}{5} \), 21. \( \frac{3}{4} \)
Changing Customary Units

Customary units of weight are the **ounce**, **pound**, and **ton**.
1 pound (lb) = 16 ounces (oz)
1 ton (T) = 2,000 pounds

Customary units of liquid capacity are the **cup**, **pint**, **quart**, and **gallon**.
1 cup (c) = 8 fluid ounces (fl oz)
1 pint (pt) = 2 cups
1 quart (qt) = 2 pints
1 gallon (gal) = 4 quarts

| Converting Units | • When you change from a larger unit to a smaller unit, multiply. There will be a greater number of smaller units than larger units. • When you change from a smaller unit to a larger unit, divide. There will be fewer larger units than smaller units. |

**EXAMPLES**

**A** Change 5 cups to pints.
*Cup is a smaller unit than pint. Divide.*

\[
5 \div 2 = 2 \frac{1}{2}
\]

5 cups = 2 \(\frac{1}{2}\) pints

**B** Change 32 pounds to ounces.
*Pound is larger than ounce. Multiply.*

\[
32 \times 16 = 512
\]

32 pounds = 512 ounces

**Try These Together**

1. 8 qt = ? gal

*HINT: Quart is smaller than gallon.*

2. \(2\frac{1}{2}\) c = ? fl oz

*HINT: Cup is larger than fluid ounce.*

**Complete.**

3. \(3\frac{2}{5}\) T = ? lb

4. 5 qt = ? pt

5. 8 lb = ? oz

6. 8,000 lb = ? T

7. \(5\frac{1}{4}\) gal = ? qt

8. 6 pt = ? qt

9. \(5\frac{3}{4}\) pt = ? c

10. 12 qt = ? gal

11. 16 fl oz = ? c

12. **Space Exploration** During liftoff, the space shuttle’s three main engines each use 1,000 lbs of fuel every second. How many tons of fuel do the three engines use together in one second?

13. **Food** A fast food restaurant sells 16-ounce drinks. How many cups are in a 16-ounce drink?

14. **Standardized Test Practice** Mauri’s little sister weighed exactly 7 pounds when she was born. How many ounces did she weigh?

   A 70       B 56       C 112       D 23

   **Answers:** 1. 2  2. 8  3. 6  4. 10  5. 10  6. 12  7. 2  8. 3  9. 5  10. 1.5  11. 2  12. 8  13. 3  14. 7  15. 1  16. 4  17. 10  18. 5  19. 8  20. 12  21. 1.5  22. 3  23. 7  24. 1  25. 4  26. 10  27. 12  28. 8  29. 3  30. 1.5  31. 2  32. 8  33. 3  34. 7
The distance around a geometric figure is called its perimeter. The area \((A)\) of a closed figure is the number of square units needed to cover its surface.

**Finding the Perimeter of a Rectangle**
The perimeter of a rectangle is the sum of the measures of the sides. It can also be expressed as 2 times the length \((\ell)\) plus 2 times the width \((w)\).

\[
P = \ell + w + \ell + w \quad \text{or} \quad P = 2\ell + 2w
\]

**Area of Rectangles**
The area \((A)\) of a rectangle equals the product of its length \((\ell)\) and width \((w)\).

\[
A = \ell w
\]

**Examples**

**A** Find the perimeter of a rectangle with a length of 4 yards and a width of 3 yards.

\[
P = 2\ell + 2w
\]

\[
P = 2(4) + 2(3) \quad \text{Replace} \quad \ell \quad \text{with} \quad 4 \quad \text{and} \quad w \quad \text{with} \quad 3.
\]

\[
P = 14 \text{ yd}
\]

**B** Find the area of a rectangle with a length of 20 cm and a width of 4 cm.

\[
A = \ell w
\]

Write the formula for area.

\[
A = \ell w
\]

\[
A = 20 \times 4 \quad \text{Replace} \quad \ell \quad \text{with} \quad 20 \quad \text{and} \quad w \quad \text{with} \quad 4.
\]

\[
A = 80 \quad \text{Multiply.}
\]

The area is 80 cm\(^2\).

**Try These Together**

1. Find the perimeter of the figure.

   \[
   \begin{align*}
   \text{length:} & \quad 3 \text{ m} \\
   \text{width:} & \quad 5 \text{ m}
   \end{align*}
   \]

   HINT: Use the formula for the perimeter of a rectangle.

2. Find the area of a rectangle with a length of 15 in. and a width of 12 in.

   HINT: The area of a rectangle is length times width.

**Practice**

Find the perimeter and area of each rectangle.

3. \[
\begin{array}{c}
\text{length:} \quad 6 \text{ ft} \\
\text{width:} \quad 2 \text{ ft}
\end{array}
\]

4. \[
\begin{array}{c}
\text{length:} \quad 7 \text{ cm} \\
\text{width:} \quad 3 \text{ cm}
\end{array}
\]

5. \[
\begin{array}{c}
\text{length:} \quad 6 \text{ cm} \\
\text{width:} \quad 4 \text{ cm}
\end{array}
\]

6. rectangle: \(\ell = 12\) inches \(w = 4\) inches

7. rectangle: \(\ell = 3\) cm \(w = 2\) cm

8. rectangle: \(\ell = 8\) feet \(w = 5\) feet

9. **Standardized Test Practice** The Angtuaco family is putting sod in their backyard. Their backyard is in the shape of a rectangle with a width of 60 feet and a length of 100 feet. How many square feet of sod will they need to cover their backyard?

A 120 ft\(^2\)  B 600 ft\(^2\)  C 6,000 ft\(^2\)  D 1,200 ft\(^2\)

ANSWERS: 1. 16 in. 2. 180 cm 3. 16 ft 4. 12 in. 5. 20 cm, 21 cm 6. 62 in. 48 in. 7. 10 cm, 6 cm
A circle is the set of all points in a plane that are the same distance from a given point called the center. The diameter \((d)\) is the distance across the circle through its center. The radius \((r)\) is the distance from the center to any point on the circle. The circumference \((C)\) is the distance around the circle.

The circumference of a circle is equal to \(\pi\) times its diameter or \(\pi\) times twice the radius.

\[
C = \pi d \\
C = 2\pi r
\]

You can use 3.14 or \(\frac{22}{7}\) as approximate values for \(\pi\).

**EXAMPLES**

**A** Find the circumference of a circle with a diameter of 9 inches.

\[
C = \pi d \\
C = 3.14 \times 9 \\
C = 28.26 \text{ inches}
\]

**B** Find the circumference of a circle with a radius of 5 feet.

\[
C = 2\pi r \\
C = 2 \times 3.14 \times 5 \\
C = 31.4 \text{ feet}
\]

**Try These Together**

Find the circumference of each circle.

1. \[3 \text{ in.} \]  
   HINT: Replace \(r\) with 3.

2. \[10 \text{ ft} \]  
   HINT: Replace \(d\) with 10.

**PRACTICE**

Find the circumference of each circle. Use 3.14 or \(\frac{22}{7}\) for \(\pi\). Round to the nearest tenth if necessary.

3. \(d = 18 \text{ cm}\)  
4. \(d = 24 \text{ m}\)  
5. \(r = 7 \text{ in.}\)  
6. \(r = 4 \text{ ft}\)

7. Recreation A plastic disc for throwing through the air has a diameter of 12 inches. What is its circumference?

8. **Standardized Test Practice** Find the circumference of a circle whose radius is 15 centimeters.

   A 47.1 cm  
   B 94.2 cm  
   C 23.6 cm  
   D 65.3 cm

**Answers:** 1. 18.8 in.  
2. 31.4 ft  
3. 66.5 cm  
4. 75.4 in.  
5. 37.7 ft  
6. 18.8 in.  
8. B
Educated Guess

Have you ever been to a carnival and seen a booth where the worker guesses people’s weight or height? Follow the steps in this activity to see if you can guess the heights of the members of your family.

1. With a parent and a tape measure, find the height of a wall in your home. Make sure the wall has lots of things on it, such as paintings, decorations, a window, and so on. Record the height of your wall in feet.

2. Find objects on your wall that are located at different fractions of the height of the wall. For example, find something, perhaps a light switch, that is located at \( \frac{1}{2} \) the height of the wall. Find another object, perhaps the top of a poster, that is located at \( \frac{3}{4} \) the height of the wall. If your wall is 8 feet tall, then \( \frac{1}{2} \) of 8 feet is 4 feet, and \( \frac{3}{4} \) of 8 feet is 6 feet. Record the heights of the different objects on your wall here.

3. Now you are ready to guess someone’s height. Have a family member walk by the wall that you measured while you are on the other side of the room. Use the objects that you measured on the wall to estimate your family member’s height. Repeat this for other family members and friends. Record the name and height of each person here.

4. Once you have guessed everyone’s height, have your parent help you measure everyone’s height using the tape measure. How close were your guesses?

Answers are located on page 107.
A ratio is a comparison of two numbers by division.

### Ratio

<table>
<thead>
<tr>
<th>Arithmetic: 5 to 1</th>
<th>5:1</th>
<th>5/1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algebra: a to b</td>
<td>a:b</td>
<td>a/b</td>
</tr>
<tr>
<td>When you write a ratio as a fraction, write it in simplest form.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Two ratios that have the same value are equivalent ratios.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>You can also write ratios as decimals. 1/4 = 1 ÷ 4, or 0.25</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Examples

**A** Alfredo gets a hit 2 out of every 10 times he goes to bat. Write this ratio as a fraction in simplest form.

\[
\frac{\text{hits}}{\text{at bats}} = \frac{2}{10} = \frac{2 ÷ 2}{10 ÷ 2} = \frac{1}{5}
\]

The GCF of 2 and 10 is 2.

**B** Dan is 66 inches tall. Joaquin is 6 feet tall. Write the ratio of Dan’s height to Joaquin’s height in simplest form.

\[
\frac{66 \text{ in.}}{6 \text{ ft}} = \frac{66 \text{ in.}}{72 \text{ in.}} = \frac{66 ÷ 6}{72 ÷ 6} = \frac{11}{12}
\]

Write both measurements in inches. 6 ft = 72 in.

The GCF of 66 and 72 is 6.

### Try These Together

**Express each ratio as a fraction in simplest form.**

1. \(\frac{12}{15}\)
   - HINT: Find the GCF and then simplify.

2. 24 to 36
   - HINT: Write as a fraction. Find the GCF and simplify.

### Practice

**Express each ratio as a fraction in simplest form.**

3. 36:9
4. 45:5
5. 33 to 3
6. 8 to 32
7. 80:10
8. 15 to 45

**Tell whether the ratios are equivalent.**

9. 72:8 and 18:2
10. 48:6 and 52:4
11. \(\frac{60}{12}\) and \(\frac{75}{5}\)
12. \(\frac{8}{24}\) and \(\frac{9}{27}\)

13. **Standardized Test Practice** Which of the following is 85 tulips to 60 petunias written as a fraction in simplest form?

   A \(\frac{12}{17}\)  
   B \(\frac{17}{5}\)  
   C \(\frac{15}{17}\)  
   D \(\frac{17}{12}\)
A rate is a ratio of two measurements with different units. A unit rate is a rate in which the denominator is 1 unit.

**Examples**

A In a bike race, Tariq rode 42 km in 2 hours. What was his unit rate?

Write the rate as a fraction. Then find the equivalent rate with a denominator of 1.

\[
\frac{\text{km}}{\text{hr}} = \frac{42}{2} = \frac{42 \div 2}{2 \div 2} = \frac{21}{1}
\]

The GCF of 42 and 2 is 2.

Tariq rode at a rate of 21 km per hour.

B Population density is the number of people per square mile. What is the population density of a town with a population of 5,250 and an area of 5 square miles?

\[
\frac{\text{5,250 people}}{\text{5 sq. mi}} = \frac{5,250}{5} \quad \text{Divide.}
\]

\[
= 1,050 \text{ people per square mile}
\]

**Try These Together**

Express each rate as a unit rate.

1. $50 for 10 days

   HINT: Write as a fraction. Then find the equivalent rate with a denominator of 1.

2. 8 revolutions in 2 minutes

   HINT: Write as a fraction. Then find the equivalent rate with a denominator of 1.

**Practice**

Express each rate as a unit rate.

3. 300 tourists in 4 days

4. 720 miles in 8 days

5. 512 yards in 8 minutes

6. $1.98 for 2 ounces

7. $22.32 for 18 gallons

8. 240 miles in 3 hours

9. 16 books in 4 days

10. 35 people in 5 vans

11. Sales A company sold 3,000 popcorn poppers last year. On average, how many popcorn poppers did they sell each month?

12. Standardized Test Practice Which of the following gas stations sells gas for the best price per gallon?

   A $18.75 for 15 gallons

   B $16.64 for 13 gallons

   C $26.00 for 20 gallons

   D $19.68 for 16 gallons
You can show that two ratios are equivalent with an equation called a proportion. When two ratios form a proportion, the cross products are equal. You can solve a proportion by using cross products to find a missing term.

**EXAMPLES**

A  Can you form a proportion with the ratios \( \frac{1}{2} \) and \( \frac{5}{10} \)?

\[
\frac{1}{2} = \frac{5}{10} \\
\text{Set the ratios equal to each other.}
\]

\[
\frac{1}{2} \times 10 = 1 \times 10 \quad \text{and} \quad \frac{5}{10} \times 2 = 2 \times 5 = 10
\]

Since the cross products are equal, the ratios form a proportion.

B  Solve \( \frac{u}{64} = \frac{3}{16} \).

\[
\frac{u}{64} = \frac{3}{16} \\
\text{Find the cross products.}
\]

\[
16u = 192
\]

\[
\frac{16u}{16} = \frac{192}{16}
\]

\[
u = 12
\]

The solution is 12.

**Try These Together**

**Solve each proportion.**

1. \( \frac{2}{3} = \frac{x}{9} \)  
   **Hint:** \( 2 \times 9 = 3x \).

2. \( \frac{3}{y} = \frac{1}{4} \)  
   **Hint:** Find the cross products.

**Solve each proportion.**

3. \( \frac{10}{16} = \frac{5}{n} \)

4. \( \frac{j}{2} = \frac{1}{4} \)

5. \( \frac{8}{6} = \frac{p}{3} \)

6. \( \frac{15}{18} = \frac{5}{k} \)

7. \( \frac{z}{40} = \frac{8}{80} \)

8. \( \frac{6}{12} = \frac{3}{f} \)

9. \( \frac{7}{3} = \frac{q}{21} \)

10. \( \frac{r}{6} = \frac{5}{30} \)

11. Find the value of \( x \) that makes \( \frac{x}{15} = \frac{8}{20} \) a proportion.

12. **Surveys**  A survey at Lincoln Middle School found that 6 of every 10 students prefer math class to science class. If there are 400 students at the school, how many of them would you expect to prefer math class to science class?

13. **Standardized Test Practice**  A factory can produce 1,500 cans of juice in 3 hours. How many cans of juice can they produce in 8 hours?

   **A** 2,000  **B** 4,000  **C** 8,000  **D** 6,000
You can use a **scale drawing** to represent something that is too large or too small for an actual-size drawing. A map is an example of a scale drawing. The **scale** on a map is the ratio of the distance on the map to the actual distance.

### EXAMPLES

**A**  Tracy and Tyrone are planning a hiking trip. On the map, their route is 7.5 cm long. The map scale says that 1 cm represents 3 km. What is the actual length of their hike?

Let \( d \) represent the hiking distance. Write and solve a proportion.

\[
\frac{\text{map distance}}{\text{actual distance}} \rightarrow \frac{1 \text{ cm}}{3 \text{ km}} = \frac{7.5 \text{ cm}}{d}
\]

\[
1 \times d = 3 \times 7.5
\]

\[
d = 22.5 \text{ km}
\]

The actual length is 22.5 km.

**B**  The scale of a blueprint is 1 in. = 4 ft. If the actual width of a porch is 16 ft, what is the width on the blueprint?

Let \( w \) represent the porch width. Write and solve a proportion.

\[
\frac{\text{blueprint width}}{\text{actual width}} \rightarrow \frac{1 \text{ in.}}{4 \text{ ft}} = \frac{w}{16 \text{ ft}}
\]

\[
1 \times 16 = 4 \times w
\]

\[
16 = 4w
\]

\[
w = \frac{16}{4} = 4
\]

The width on the blueprint is 4 inches.

### Try These Together

**Find the length of each object on a scale drawing with the given scale.**

1. a house 75 feet long; 1 inch:1 foot
2. a box 3 meters tall; 1 cm:1.5 m

### Practice

**Find the length of each object on a scale drawing with the given scale.**

3. a desk 4.5 meters long; 2 centimeters:1 meter
4. an airplane with a 54-meter wingspan; 3 centimeters:1 meter
5. an automobile that is 8 feet wide; 0.5 inch:1 foot
6. a street that is 2 miles long; 5 inches:1 mile
7. Find how far it is across the city of Bloomington if it is 2.45 centimeters on a map that has a scale of 1 centimeter : 3 kilometers.
8. **Architecture**  The Sears Tower in downtown Chicago is 110 stories high. A scale drawing has a scale of 1.5 centimeters:1 story. How tall is the Sears Tower on the scale drawing?

9. **Standardized Test Practice**  Which is the actual length of a sofa that is 4 inches on a scale drawing if the scale is 2 inches:5 feet?

| A | 10 feet |
| B | 20 feet |
| C | 15 feet |
| D | 30 feet |
You can use a proportion to express a fraction as a percent. To write a percent as a fraction, begin with a fraction that has a denominator of 100. Then write the fraction in simplest form. Recall that \( \frac{n}{100} = n\% \).

**EXAMPLES**

**A** Write \( \frac{4}{5} \) as a percent.

\[
\frac{4}{5} = \frac{n}{100} \quad \text{Write a proportion}
\]

400 = 5n  
Multiply to find the cross products.

\[
\frac{400}{5} = \frac{5n}{5} \quad \text{Divide each side by 5.}
\]

80 = n

\[
\frac{4}{5} = 80\%
\]

**B** Write 24% as a fraction in simplest form.

Estimate: 24% is about 25%, which is \( \frac{1}{4} \).

\[
24\% = \frac{24}{100}
\]

\[
= \frac{24 \div 4}{100 \div 4} \quad \text{The GCF is 4.}
\]

\[
= \frac{6}{25}
\]

24% = \( \frac{6}{25} \)  
Compare to the estimate.

**Try These Together**

Write each fraction as a percent.

1. \( \frac{2}{5} \)  
2. \( \frac{1}{2} \)  

**HINT:** Solve a proportion in which the fraction is equal to \( \frac{n}{100} \).

**Practice**

Write each fraction as a percent.

3. \( \frac{3}{10} \)  
4. \( \frac{3}{4} \)  
5. \( \frac{2}{6} \)  
6. \( \frac{15}{20} \)  
7. \( \frac{4}{16} \)  
8. \( \frac{3}{5} \)

Write each percent as a fraction in simplest form.

9. 20%  
10. 85%  
11. 25%  
12. 62.5%  
13. 87.5%  
14. 30%  
15. 15%  
16. 37.5%

17. **Hobbies** At a hot air balloon festival, 60% of the hot air balloons were partially colored red. Write the percent of hot air balloons that were partially colored red as a fraction in simplest form.

18. **Standardized Test Practice** A store advertised a sale where every item was \( \frac{1}{8} \) off. What is this fraction written as a percent?

A 2.5%  
B 12.5%  
C 22.5%  
D 37.5%
When you express a percent greater than 100% as a decimal, the resulting decimal is greater than 1. When you express a percent less than 1% as a decimal, the resulting decimal is less than 0.01.

**EXAMPLES**

A. Write 142% as a decimal.
   
   \[ 142\% = 1.42 \]
   
   Divide the percent by 100 and remove the percent symbol.

B. Write 0.00825 as a percent.
   
   \[ 0.00825 = 0.825\% \]
   
   Multiply the decimal by 100 and add the percent symbol.

**Try These Together**

1. Express 0.682% as a decimal.
   
   **HINT:** The resulting decimal is less than 0.01.

2. Express 3.7 as a percent.
   
   **HINT:** The decimal is greater than 1.

**Practice**

Express each percent as a decimal.

3. 125%
4. 545%
5. 210%
6. 356%
7. 0.08%
8. 0.85%

Express each number as a percent.

9. 7
10. 0.007
11. 1.28
12. 4.5
13. 3.86
14. \[ \frac{8}{2,000} \]

15. Write 430% as a decimal.
16. Express 0.006 as a percent.

17. Internet Lara and Lezlie both use the Internet. Lara’s Internet usage is 160% of Lezlie’s. By what decimal would you multiply Lezlie’s usage to get Lara’s usage?

18. **Standardized Test Practice** The area of Indiana is about 0.0098 of the area of the United States. What is 0.0098 written as a percent?

   A 0.98%    B 98%    C 9.8%    D 0.0098%
Suppose you read in the school newspaper that 62% of students who were surveyed buy their lunches at school. Of the 200 students who were surveyed, how many buy their lunches? You can either use a proportion or multiplication to solve this problem.

**EXAMPLES**

**A** Find 62% of 200.

*Method 1: Use a proportion.* Let \( s \) represent the number of students who buy their lunches.

\[
\frac{s}{200} = \frac{62}{100}
\]

The ratio of \( s \) students to the 200 who were surveyed equals 62%, or \( \frac{62}{100} \).

\[
s \times 100 = 200 \times 62
\]

Find the cross products.

\[
\frac{100s}{100} = \frac{12,400}{100}
\]

Divide each side by 100.

\[
s = 124
\]

So, 62% of 200 is 124.

**B** Find 62% of 200.

*Method 2: Use multiplication.*

First express the percent as a decimal and then multiply.

\[
62\% \text{ of } 200 = 0.62 \times 200
\]

Remember, \( 62\% = 0.62 \).

\[
= 124
\]

So, 62% of 200 is 124.

**Try These Together**

*Find each number. Round to the nearest tenth if necessary.*

1. Find 50% of 66.

2. What number is 20% of 200?

3. Find 80% of 40.

4. What number is 75% of 24?

5. What number is 12% of 68?

6. 85% of 225 is what number?

7. Find 25% of 160.

8. Find 33% of 90.

9. 15% of 74 is what number?

10. What number is 37% of 128?

11. What number is 18% of 96?

12. Find 10% of 100.

13. **Landscaping** Mr. and Mrs. Morrisey want to replace 60% of the grass in their yard with bushes and plants. If their yard has 6,000 square feet of grass, how many square feet will be replaced with bushes and plants?

14. **Standardized Test Practice** If 25% of the pieces of a 500-piece puzzle are edge pieces, how many are edge pieces?

A 75  B 150  C 125  D 225
The percent proportion is \( \frac{a}{b} = \frac{p}{100} \), where \( a \) represents the part, \( b \) represents the base, and \( p \) represents the percent. Using the proportion \( \frac{1}{2} = \frac{50}{100} \), you can see how to use the percent proportion to solve the three basic types of percent equations.

### EXAMPLES

**A** 32 is 40% of what number?

\[
\frac{32}{b} = \frac{40}{100}
\]

Write the percent proportion replacing \( a \) with 32 and \( p \) with 40.

\[
32 \times 100 = b \times 40
\]

Find the cross products.

\[
\frac{3.200}{40} = \frac{40b}{40}
\]

Divide each side by 40.

\[80 = b\]

So, 32 is 40% of 80.

**B** 30 is what percent of 150?

\[
\frac{30}{150} = \frac{p}{100}
\]

Write the percent proportion replacing \( a \) with 30 and \( b \) with 150.

\[
30 \times 100 = 150 \times p
\]

Find the cross products.

\[
\frac{3.000}{150} = \frac{150p}{150}
\]

Divide each side by 150.

\[20 = p\]

So, 30 is 20% of 150.

### Try These Together

**Find each number. Round to the nearest tenth if necessary.**

1. 6 is what percent of 12?
2. What number is 68% of 90?

**Find each number. Round to the nearest tenth if necessary.**

3. What percent of 24 is 6?
4. What percent of 40 is 8?
5. 8% of what number is 10?
6. 15 is 30% of what number?
7. What number is 60% of 20?
8. What percent of 96 is 12?
9. What percent of 400 is 60?
10. What number is 12% of 50?
11. Find 110% of 16.
12. 90 is 45% of what number?

13. **Standardized Test Practice** During a flu epidemic, 3 of the 20 students in Marina’s class were absent with the flu. What percent of the students in Marina’s class were absent?

   - A 10%
   - B 5%
   - C 20%
   - D 15%

**Answers:** 3. 61.2 4. 20% 6. 12 7. 9 10. 6 11. 16 12. 15 13. D
Chapter 7 Review

Commercial Percents
Television stations and networks sell advertising time, called commercials, to pay for the cost of running the station and network, and to pay for producing the programs that you watch. Have you ever wondered how much time you spend watching commercials instead of programs? With a parent, follow the steps below to find out. Note: If you or your parent would rather not watch television, you can complete this same exercise by listening to the radio.

1. Decide on a television program to watch. Before the program starts, have this worksheet, a pencil, and a watch with a second hand ready.

2. Using the table below, record the exact time the program begins. Then, using the watch, record the length of time of every commercial break during the program. Networks and television stations usually show four or more commercials in a row, so be sure to record the length of time of each entire commercial break. Also be sure to record the length of time of the commercials after the program, right up until the next program begins.

3. Once you have watched the entire program, find the percent of time you spent watching commercials and the percent of time you spent watching the program. Use the table below to help you.

Use the percent proportion to find out what percent of the program’s time was spent on commercials. The total time (in minutes) of the commercials is the percentage ($P$) and the total program time is the base ($B$). You are looking for $r$. Round your result to the nearest whole percent.

\[
\frac{P}{B} = \frac{r}{100}
\]

Percentage of time for commercials:

Try this activity with different kinds of programs, with programs of different lengths, and with programs shown at different times of the day. Are the percentages the same? Explain.

Answers are located on page 107.
The following examples show two different ways to estimate percents.

**EXAMPLES**

**A** Estimate 21% of 196.
Round 21% to 20% and 196 to 200. Use a fraction.

20% is the same as \( \frac{1}{5} \).

\( \frac{1}{5} \) of 200 is 40.

21% of 196 is about 40.

**B** Estimate 21% of 196.
Round 21% to 20% and 196 to 200. Find 10% and multiply.

10% is the same as \( \frac{1}{10} \) or 0.1.

10% of 200 is 0.1(200) or 20.
Now find 20% or 2 times 10% of 200.
2 × 20 = 40
21% of 196 is about 40.

**Try These Together**

**Estimate.**

1. 32% of 87.5
2. 51% of 520
3. 81% of 49

**HINT:** Round the number and the percent. Then use one of the methods from the examples.

**Estimate the percent shaded. Then count to find the exact percent.**

4. 
5. 
6. 

**Write the fraction, decimal, mixed number, or whole number equivalent of each percent that could be used to estimate.**

7. 18%  
8. 0.9%  
9. 25.54%  
10. 400%  
11. 75%  
12. \( \frac{9}{10} \)%

**Estimate.**

13. 48% of 139  
14. 9% of 12  
15. 73.5% of 61  
16. 9% of 122  
17. 153% of 21  
18. 0.9% of 810

19. **Nutrition** Based on a 2,000 Calorie diet, the recommended daily allowance (R.D.A.) for fat is 65 grams. One serving of whole milk yogurt has 11% of the R.D.A. for fat. About how many grams is that?

20. **Standardized Test Practice** Estimate 250% of 39.

A 10  
B 100  
C 120  
D 200

Sample answers are given. 1. 27  
2. 250  
3. 4  
4. 10.4  
5. 30%  
6. 40%  
7. 19.8  
8. 19.  
9. 7.2  
10. 6.  
11. 0.3  
12. 0.2  
13. 1.  
14. 2.7  
15. 4.  
16. 0.1  
17. 0.4  
18. 0.01  
19. 0.001  
20. 0.0001

Sample answers are given.
In Chapter 7, you learned that you can solve many percent problems with the percent proportion, \( \frac{a}{b} = \frac{p}{100} \). Remember that \( a \) is the part, \( b \) is the base, and \( p \) is the percent. The rule below is the percent proportion written as an equation. You can use the percent equation to solve percent problems.

\[
\text{Percent Equation: } \quad \frac{\text{part}}{\text{base}} = \frac{\text{percent}}{100} = \frac{\text{part}}{\text{base}}
\]

### EXAMPLES

**A** What number is 45% of 72?

- 45% or 0.45 is the percent and 72 is the base.
- Let \( n \) represent the part.
- \( \frac{\text{part}}{\text{base}} = \frac{\text{percent}}{100} \)
- \( n = 0.45 \cdot 72 \)
- \( n = 32.4 \)

**B** 24 is 80% of what number?

- 24 is the part and 80% or 0.8 is the percent.
- Let \( n \) represent the base.
- \( \frac{\text{part}}{\text{base}} = \frac{\text{percent}}{100} \)
- \( 24 = 0.8 \cdot n \)
- \( \frac{24}{0.8} = \frac{0.8n}{0.8} \)
- \( 30 = n \)

The base is 30.

### Try These Together

**Use a percent equation to solve each problem. Round to the nearest tenth if necessary.**

1. 29 is what percent of 75?
   - *HINT: The number following “of” is usually b.*
2. Find 73% of 147.
   - *HINT: The number with the % is the percent.*

### PRACTICE

**Use a percent equation to solve each problem. Round answers to the nearest tenth.**

3. Find 70% of 49.
4. 33% of what number is 1.048?
5. What percent of 97 is 39?
6. 47.7% of what number is 70?
7. 24% of 16 is what number?
8. 24 is 31% of what number?
9. 19 is what percent of 14?
10. 79 is 60% of what number?
11. 15% of 64 is what number?
12. 20 is what percent of 400?
13. 71 is what percent of 23?
14. Find 82% of 84.

15. **Standardized Test Practice** What is 2.1% of 76? Round to the nearest tenth.
   - A 1.6  B 3.6  C 15.0  D 159.6
If you want to make a prediction about a large group of people, you may wish to use a smaller group, or sample, from the larger group. The large group from which you gathered your sample is known as the population. To make sure your information represents the population, the sample must be drawn at random. A random sample gives everyone the same chance of being selected.

### Practice

1. **Music** Oakdale Middle School’s school newspaper surveyed the school’s students by asking them what their favorite type of music is. The school has 1,020 students.
   a. What was the sample size?
   b. To the nearest percent, what percent of students preferred pop/rock?
   c. How many students in the school would you expect to say that pop/rock is their favorite?
   d. If the school newspaper had only surveyed students from some boys’ physical education classes, would that be a random sample? Explain.

2. **Sales** Each month Peterman’s Books, a local bookstore, randomly surveys customer purchases for marketing purposes.
   a. Do you think this sample is representative of every bookstore in the United States? Why or why not?
   b. Of the 482 books sold on Tuesday, how many would you expect to be romance books?

#### March Survey

<table>
<thead>
<tr>
<th>Books</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fiction</td>
<td>31%</td>
</tr>
<tr>
<td>Non-Fiction</td>
<td>26%</td>
</tr>
<tr>
<td>Children’s</td>
<td>10%</td>
</tr>
<tr>
<td>Romance</td>
<td>8%</td>
</tr>
<tr>
<td>Mystery</td>
<td>5%</td>
</tr>
<tr>
<td>Horror/Science Fiction</td>
<td>4%</td>
</tr>
<tr>
<td>Other</td>
<td>16%</td>
</tr>
</tbody>
</table>

#### Type of Music | Number
- Pop/Rock       | 58
- Rap/Hip Hop    | 37
- Country        | 32
- R & B          | 15
- Other          | 8
Percent of Change describes how much a quantity increases or decreases.

**Example**

Last year, Melvin paid $12 to purchase a ticket to a water amusement park. This year, the ticket cost him $14. What was the percent of increase?

**Step 1** Find the amount of increase. $14 - $12 = $2

**Step 2** Use the percent proportion. \[
\frac{\text{amount of increase}}{\text{original amount}} = \frac{r}{100}
\]

\[
\frac{2}{12} = \frac{r}{100}
\]

**Step 3** Solve for \(r\). \(2 \cdot 100 = 12r\) Find the cross products.

\[
\frac{200}{12} = \frac{12r}{12}
\]

Divide each side by 12.

\[
16.7 = r
\]

The percent of increase is about 17%.

**Try These Together**

*Find the percent of change. Round to the nearest whole percent.*

1. original: $5  
   new: $2

2. original: $10  
   new: $7

3. original: 60  
   new: 54

*HINT: Find the amount of change, then set up the percent proportion and solve for \(r\).*

**Practice**

*Find the percent of change. Round to the nearest whole percent.*

4. original: 27.5  
   new: 35.5

5. original: $186  
   new: $196

6. original: 64  
   new: 70

7. original: $3  
   new: $6

8. original: 34  
   new: 59

9. original: $77  
   new: $110

10. original: 50  
    new: 63

11. original: $41.50  
    new: $10.50

12. original: 93  
    new: 19

13. original: $61  
    new: $72

14. original: $38  
    new: $49

15. original: 67  
    new: 55

16. Money Matters Hank pays $580 each month for rent. Next month his rent increases to $620. What is the percent of increase?

17. Standardized Test Practice What is the percent of increase from 50 to 75?

   A 25%  
   B 33%  
   C 50%  
   D 75%
You can use one of the two methods that follow to find the total cost of an item including sales tax or the sale price of an item including a discount.

**Sales Tax and Discount**

<table>
<thead>
<tr>
<th><strong>Sales Tax</strong></th>
<th><strong>Discount</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Method 1</strong></td>
<td><strong>Method 2</strong></td>
</tr>
<tr>
<td>• First, find the amount of the sales tax, ( t ).</td>
<td></td>
</tr>
<tr>
<td>• Then add the sales tax to the price of the item.</td>
<td></td>
</tr>
<tr>
<td>• First, subtract the percent of discount from 100%.</td>
<td></td>
</tr>
<tr>
<td>• Then multiply to find the sale price including discount.</td>
<td></td>
</tr>
</tbody>
</table>

**EXAMPLES**

**A** Find the total cost of a $72 item with 6% sales tax.
6% of $72 = \( t \), or $4.32
$72 + $4.32 = $76.32
The total cost is $76.32.

**B** Find the sale price of a $250 item with a 30% discount.
100% - 30% = 70%
So the sale price will be 70% of the original price.
$250 \times 0.7 = $175
The sale price is $175.

**Try These Together**

**Find the total cost or sale price to the nearest cent.**

1. $6.95 lamp; 15% off
   
   HINT: The sale price will be less than $6.95.

2. $19.50 sweatpants; 6% tax
   
   HINT: The total cost will be more than $19.50.

3. $24.50 sandals; 20% discount

4. $230 trampoline; 30% off

5. $25 dartboard; 6 \( \frac{1}{2} \) % tax

6. $37.95 computer game; 5 \( \frac{1}{2} \) % tax

7. **Standardized Test Practice**
   What is the sale price to the nearest cent of a $79 rug on sale for 10% off?
   
   **A** $86.90
   
   **B** $78.21
   
   **C** $71.10
   
   **D** $69.00
Simple interest is money you pay the bank or lender for the use of money. Similarly, if you deposit money in a savings account, the bank may pay you simple interest for the use of the money.

### Simple Interest

The formula for simple interest is \( I = \text{principal} \times \text{rate} \times \text{time} \), where \( I \) is the interest, \( p \) is the principal, or the amount of money invested or borrowed, \( r \) is the annual interest rate, and \( t \) is the time, in years.

### Examples

**A** What interest would you earn on a savings deposit of $1,200 at 8% interest for six months?

\[
I = \text{principal} \times \text{rate} \times \text{time} \\
I = 1,200 \times 0.08 \times \frac{1}{2} \\
p = $1,200, r = 8\% \text{ or } 0.08, t = 6 \text{ months or } \frac{1}{2} \text{ year} \\
I = 48 \text{ } \\
\text{The interest you would earn is$48.}
\]

**B** How much interest would you pay on a credit card balance of $2,000 at 15% interest for 1 year?

\[
I = \text{principal} \times \text{rate} \times \text{time} \\
I = 2,000 \times 0.15 \times 1 \\
p = $2,000, r = 15\% \text{ or } 0.15, t = 1 \text{ year} \\
I = 300 \text{ } \\
\text{The interest you would pay is$300.}
\]

### Try These Together

**Find the interest to the nearest cent for each principal, interest rate, and time.**

1. $420, 9%, 6 months
   
   **HINT:** \( p = 420; r = 0.09; t = \frac{1}{2} \)

2. $816, 7%, 9 months
   
   **HINT:** Replace \( p, r, \) and \( t \) with the values given.

3. $3,800, 10%, 1 year

4. $2,903, 11%, 18 months

5. $850.30, 3.75%, 1 year

6. $283.85, 8.5%, 2 years

### Practice

**Find the interest to the nearest cent for each principal, interest rate, and time.**

3. $3,800, 10%, 1 year

4. $2,903, 11%, 18 months

5. $850.30, 3.75%, 1 year

6. $283.85, 8.5%, 2 years

**Find the interest to the nearest cent on credit cards for each credit card balance, interest rate, and time.**

7. $844, 9%, 3 years

8. $3,000, 12%, 3 months

9. $1,700, 24%, 9 months

10. $275, 17.5%, 2 years

11. **Standardized Test Practice**

   What is the interest on a credit card balance of $500 over two years if the interest rate is \( 10\frac{1}{2}\% \)?

   **A** $150.00  
   **B** $105.00  
   **C** $102.00  
   **D** $52.50
Which Price is Right?
Cut out the cards below and shuffle them. Give three cards to your parent and three to yourself. Then, pick the item on each card with the lower price. You each get one point for each correct answer. The one with the most points wins the round. Exchange cards and play again, or create your own cards.

1. **A.** A pair of in-line skates with an original cost of $60 after a 20% discount  
   **B.** A pair of in-line skates with an original cost of $60 after a $15 discount

2. **A.** A music CD bought from an online retailer for $12 plus $3.20 shipping and handling  
   **B.** A music CD bought from a local music store for $14 plus 8% sales tax

3. **A.** A group of five friends who go to an amusement park in the same car and pay $60 per carload for admission  
   **B.** A group of five friends who go to an amusement park and pay $15 each for admission

4. **A.** Internet service that costs $9.99 per month  
   **B.** Internet service that costs $99 per year

5. **A.** A mountain bike that costs three payments of $89  
   **B.** A mountain bike that costs $275

6. **A.** A $19 pizza plus 15% gratuity at a restaurant  
   **B.** A $20 pizza plus 8\(\frac{1}{4}\)% sales tax from a takeout window

Answers are located on page 107.
A simple **event** is a specific outcome. Outcomes occur at **random** if each outcome occurs by chance.

The **probability** of an event is a ratio that compares the number of favorable outcomes to the number of possible outcomes.

\[
P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}
\]

**Example**

A certain spinner is equally likely to stop on each of its regions labeled 5, 10, 15, 20, and 25. Find the probability that the spinner will stop on an even number.

\[
P(\text{even number}) = \frac{\text{number of ways an even number occurs}}{\text{number of possible outcomes}}
\]

Since 2 of the outcomes are even numbers (10 and 20), and there are 5 possible outcomes, \( P(\text{even number}) = \frac{2}{5} \).

**Try These Together**

1. What is the probability that a month chosen at random will have 31 days?  
   **HINT:** How many months out of 12 have 31 days?

2. What is the probability that a day of the week chosen at random has a name that starts with S?  
   **HINT:** How many days start with S?

**Practice**

A number cube for a game has six sides numbered 1–6. Find the probability that the number cube will land on each of the following when it is tossed.

3. a 2
4. a multiple of 2
5. an odd number
6. a number greater than 5

**There are 16 colored tennis balls in a bag. Three are blue, 5 are yellow, 4 are green, and 4 are orange. If you reach in the bag and draw one ball at random, what is the probability that you will draw each of the following?**

7. a green ball
8. a blue ball

9. **Standardized Test Practice**
   Ophelia is eating colored candies. There are 80 candies in all and 16 of them are red. What is the probability that she will randomly choose a red candy? Express the fraction in simplest form.

   \[
   \begin{array}{cccc}
   \text{A} & \frac{2}{10} & \text{B} & \frac{1}{5} & \text{C} & \frac{1}{10} & \text{D} & \frac{16}{80}
   \end{array}
   \]

   **Answers:** 1, 2, 3, 4.
One way to find possible outcomes and probability is with a tree diagram.

**Example**

At a concession stand, you can order a small, medium, or large cola, with or without ice. Use a tree diagram to find the number of possible outcomes.

<table>
<thead>
<tr>
<th>Ice</th>
<th>Size</th>
<th>Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>ice</td>
<td>small</td>
<td>small cola with ice</td>
</tr>
<tr>
<td></td>
<td>medium</td>
<td>medium cola with ice</td>
</tr>
<tr>
<td></td>
<td>large</td>
<td>large cola with ice</td>
</tr>
<tr>
<td>no ice</td>
<td>small</td>
<td>small cola without ice</td>
</tr>
<tr>
<td></td>
<td>medium</td>
<td>medium cola without ice</td>
</tr>
<tr>
<td></td>
<td>large</td>
<td>large cola without ice</td>
</tr>
</tbody>
</table>

**Try These Together**

For each situation, use a tree diagram to find the total number of outcomes.

1. choosing white or rye bread with either ham, turkey, or salami
2. going in-line skating or biking to either the library, grocery store, or the mall
3. buying a sweater or a shirt in either orange, blue, turquoise, or red
   *Hint: Each of the two objects in the first set goes with each of the objects in the second set.*

**Practice**

For each situation, use a tree diagram to find the total number of outcomes.

4. growing tulips, roses, or daisies in either pink, white, or yellow
5. taking a sculpture or woodworking class at either a school, a community center, or a museum
6. sitting in a room with a sofa, a chair, a love seat or a recliner, in either a soft, hard, or medium firmness
7. Music  You are in charge of music for a party. You bring three CDs: pop, jazz, and country. How many different ways can you play all three CDs so that each one is played exactly once?

**8. Standardized Test Practice**  A baseball manager has four possible starting pitchers for a game. He also must decide which of two catchers to put in the starting lineup. How many ways can he choose the players for these two positions?

\[\text{A} \quad 6 \quad \text{B} \quad 8 \quad \text{C} \quad 9 \quad \text{D} \quad 16\]
In Lesson 13-2, you learned to find outcomes using a tree diagram. In this lesson, you will learn to use the Fundamental Counting Principle to find the number of possible outcomes.

The Fundamental Counting Principle

If an event \( M \) can occur \( m \) ways and is followed by an event \( N \) that can occur \( n \) ways, then the event \( M \) followed by \( N \) can occur \( m \times n \) ways.

**Example**

Yvette can take her driving test on Monday, Wednesday, or Friday, at 4:00 P.M., 5:00 P.M., or 6:00 P.M. How many different opportunities does she have to take her driving test?

<table>
<thead>
<tr>
<th>number of days the test is given</th>
<th>number of times per day the test is given</th>
<th>opportunities to take the test</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>3</td>
<td>9</td>
</tr>
</tbody>
</table>

There are 9 opportunities for Yvette to take her driving test.

**Try These Together**

Use the Fundamental Counting Principle to find the total number of outcomes in each situation.

1. creating new hybrid flowers with short or long petals in either purple, red, or yellow
2. baking a yellow, chocolate, strawberry, or vanilla cake frosted with either vanilla, chocolate, cherry, or strawberry frosting

*Hint: Find the number of ways each event occurs, and multiply.*

**Practice**

Use the Fundamental Counting Principle to find the total number of outcomes in each situation.

3. rolling three six-sided number cubes
4. making a sandwich with either wheat or pumpernickel bread, and either salami, turkey, or pastrami, and either mustard, mayonnaise, butter, or horseradish

5. **Automobiles** Each license plate in a given state contains three letters and three numbers. What is the total number of license plates if the first three characters are letters and the last three characters are digits?

6. **Standardized Test Practice** Every Social Security card has a nine-digit identification number. How many possible Social Security numbers are there?

A 100,000  
B 1,000,000  
C 100,000,000  
D 1,000,000,000
Suppose you need to arrange 8 books on a bookshelf in the library. How many ways could you arrange the books? What you’re trying to count are permutations. You can find the answer to this question by finding $8!$ or eight factorial.

**Permutation**
A permutation is an arrangement, or listing, of objects in which order is important.

**Factorial**
The expression $n$ factorial ($n!$) is the product of all the counting numbers beginning with $n$ and counting backward to 1. For example, $3! = 3 \times 2 \times 1$, or 6.

**Example**

How many ways can you arrange 8 books on a bookshelf?

Each arrangement is a permutation. Since there are 8 books, there are eight different choices for the first book you place on the shelf. Once the first book is placed, there are seven choices for the second book, six choices for the third book, and so on.

number of permutations = $8!$
number of permutations = $8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$
number of permutations = 40,320

There are 40,320 ways to arrange the eight books.

**Try These Together**

Find the value of each expression.

1. $10 \times 9 \times 8 \times 7$
2. $5!$

**HINT:** For factorials, start with the given number and multiply by each lower number down to one.

**Practice**

Find the value of each expression.

3. $4!$
4. $6!$
5. $5 \times 4 \times 3$
6. $12 \times 11 \times 10$

7. How many ways could you and two friends line up to run a race?
8. You must select a five-digit password, where each digit must be a number from 0 to 9 without repeating any numbers. How many passwords are there?
9. Television  There are 51 contestants in a talent pageant each fall. How many ways can first place and runner-up be awarded?
10. **Standardized Test Practice**  A television network has six different prime-time slots to fill in one evening. They can choose from ten different shows. How many arrangements of programs could the network show?

**Answers:**
1. 5,040 2. 120 3. 24 4. 720 5. 60 6. 1,220 7. 6 8. 5,040 9. 2,520 10. B

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An arrangement, or listing, of objects in which order is not important is called a combination. You can find the number of combinations of objects by dividing the number of permutations of the entire set by the number of ways each smaller set can be arranged.

**Example**

How many combinations of two menu items can be chosen from a menu of four items?

There are \(4 \times 3\) permutations of two items chosen from the menu of four.

There are \(2!\) or \(2 \times 1\) ways to arrange the two items.

\[
\frac{4 \times 3}{2 \times 1} = \frac{12}{2} = 6
\]

There are 6 combinations of two menu items that can be chosen from the menu of four items.

**Try These Together**

**Solve each problem.**

1. How many ways can a three-topping pizza be made if the chef must choose from seven ingredients? Assume all three toppings are different.

2. There are four computer-programming jobs to fill from a pool of six applicants. How many ways can four programmers be chosen?

**Practice**

**Solve each problem.**

3. In how many ways can 2 flight attendants be selected from a group of 5 to work a flight?

4. Any five Supreme Court justices comprise a majority for the group of nine justices. How many groups of five are there?

5. **Standardized Test Practice** How many ways can a four-member debate team be selected from a group of eight students?

   A 60   B 70   C 80   D 90

Answers: 1. 35 2. 16 3. 10 4. 126 5. B
Theoretical probability is the expected probability of an event occurring. For example, the theoretical probability of rolling a 1 on a number cube is \( \frac{1}{6} \). That is because only one side of a number cube shows a 1, the event you are trying to get, while there are six total sides, or possible outcomes.

### Finding Theoretical Probability

\[
P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}
\]

### Experimental Probability

The experimental probability of an event is the estimated probability based on the number of positive outcomes in an experiment. To find the experimental probability of rolling a 1 on a number cube, you would roll a number cube repeatedly and record the outcomes.

A class of 32 students has 18 boys and 14 girls. If one student is chosen to take attendance for the semester, what is the probability that a boy is chosen?

\[
\text{number of ways to choose a boy} \quad \frac{18}{32} = \frac{9}{16}
\]

Therefore, \( P(\text{a boy being chosen}) = \frac{18}{32} \) or \( \frac{9}{16} \).

### Try These Together

If you have 12 coins (5 pennies, 4 nickels, 2 dimes, and 1 quarter) in a bag, find the theoretical probability of selecting:

1. one quarter in one draw.
2. one penny in one draw.

**HINT:** Think of the ratio of the number of coins in the bag that you want to draw to the total number of coins in the bag.

### Practice

Use the same situation for drawing coins as above. Find the theoretical probability of making each selection.

3. one dime in one draw
4. one nickel in one draw

5. **Standardized Test Practice** Lavon had a bag of candies. There were 20 candies in the bag: 6 red, 5 orange, 3 brown, 2 yellow, and 4 blue. Without looking, she chose a candy, recorded the color, and returned the candy to the bag. She performed this experiment 100 times and found that she chose an orange candy 22 times. What was the experimental probability of choosing an orange candy?

\[
A \quad \frac{1}{5} \quad B \quad \frac{1}{4} \quad C \quad \frac{11}{50} \quad D \quad \frac{24}{100}
\]

**Answers:** 1. \( \frac{3}{10} \) 2. \( \frac{1}{4} \) 3. \( \frac{21}{50} \) 4. \( \frac{6}{25} \)
If you roll two number cubes, the number that you roll on the second cube is not affected by the number you rolled on the first cube. These events are called independent events. If the result of one event affects the result of a second event, the events are called dependent events.

### EXAMPLES

**A** Find the probability of tossing a 5 on each of two number cubes. These are independent events.

\[
P(5 \text{ on one cube}) = \frac{1}{6} \quad \text{because there are six numbers on a cube.}
\]

\[
P(5 \text{ on each cube}) = \frac{1}{6} \times \frac{1}{6}, \text{ or } \frac{1}{36}.
\]

**B** You have four pennies and four nickels in a bag. What is the probability of drawing two pennies in a row, if you keep the first coin you draw? These two draws are dependent events.

\[
P(\text{penny on first draw}) = \frac{4}{8} \quad \text{or } \frac{1}{2} \quad \text{because there are 4 pennies and 8 coins total.}
\]

\[
P(\text{penny on second draw}) = \frac{3}{7} \quad \text{because you removed one penny, leaving 3 pennies and 7 coins total.}
\]

\[
P(\text{two pennies in a row}) = \frac{1}{2} \times \frac{3}{7} \quad \text{or } \frac{3}{14}.
\]

### Try These Together

**Tell whether each event is independent or dependent.**

1. tossing a coin twenty times
2. choosing two cards from one deck, keeping the first card.

*Hint: Does one event affect the other event?*

### Find the probability of each event.

3. tossing an even number on each of two number cubes
4. A bag contains three blue marbles, four red marbles, and two clear marbles. Three are drawn without each selection being replaced. Find \( P(\text{red, then blue, then clear}). \)

5. **Standardized Test Practice** There are 3 bottles of juice and 4 bottles of water in Nate’s ice chest. What is the probability that he will reach into the ice chest without looking and pull out two bottles of water in a row if he does not replace the first bottle?

   A \( \frac{1}{2} \) \hspace{1cm} B \( \frac{4}{7} \) \hspace{1cm} C \( \frac{2}{7} \) \hspace{1cm} D \( \frac{3}{6} \)

---

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Mathematics: Applications and Concepts, Course 2
Combination Photo Opportunity

You, your parent, and 10 other family members bought two rolls of film (with 36 exposures each) so you could take group photos during a recent trip to the beach.

You and your parent must figure out how to take group pictures while you are at the beach. Should you take pictures in groups of two, so each person will have their picture taken with every other person in the group exactly once? Will you have enough film for groups of two? Would groups of three or larger be better? Answer the questions below with your parent to find out.

1. How many groups of two family members each can be formed from the 12 members?

2. How many groups of three family members each can be formed from the 12 members?

3. How many groups of four family members each can be formed from the 12 members?

4. For which of these size groups do you have enough film?

Answers are located on page 107.
An angle is made up of two rays, or sides, with a common endpoint, or vertex. You measure angles in units called degrees. Angles are classified according to their measure.

<table>
<thead>
<tr>
<th>Types of Angles</th>
<th>Right Angle</th>
<th>Straight Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>exactly 90°</td>
<td>exactly 180°</td>
</tr>
<tr>
<td>Acute Angle</td>
<td>less than 90°</td>
<td>between 90° and 180°</td>
</tr>
<tr>
<td>Obtuse Angle</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Try These Together

Classify each angle as acute, obtuse, right, or straight.
1. 2. 3.

Classify each angle as acute, obtuse, right, or straight.
4. 5° 5. 135° 6. 90° 7. 28°
8. 9. 10.

Draw an angle having each measurement.
11. 115° 12. 30° 13. 10° 14. 160°

15. **Standardized Test Practice** Which angle is not acute?
   A 88°  B 92°  C 48°  D 65°
You can use a **circle graph** to compare parts of a whole.

### Example

Of those polled, 24% preferred Candidate A, 58% preferred Candidate B, and 18% preferred Candidate C. Express this information in a circle graph.

- **Step 1**
  - Find the number of degrees for each part of the graph.
  - Candidate A: 24% of 360° = 0.24 × 360° = 86.4°
  - Candidate B: 58% of 360° = 0.58 × 360° = 208.8°
  - Candidate C: 18% of 360° = 0.18 × 360° = 64.8°

- **Step 2**
  - Use a compass to draw a circle. Then draw a radius.

- **Step 3**
  - You can start with the least number of degrees, in this case, 64.8°. Use your protractor to draw an angle of 64.8°. Repeat this step for each part.

- **Step 4**
  - Label each section of the graph with the category and percent. Give the graph a title.

### Practice

1. **Population** Refer to the table.
   - a. Write a ratio that compares each number with the total. Write as a decimal to the nearest thousandth.
   - b. Find the number of degrees for each section of the graph. Round to the nearest tenth.
   - c. Make a circle graph of the world population.

<table>
<thead>
<tr>
<th>Region</th>
<th>Population (millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>North America</td>
<td>316</td>
</tr>
<tr>
<td>Latin America</td>
<td>525</td>
</tr>
<tr>
<td>South America</td>
<td>350</td>
</tr>
<tr>
<td>Europe</td>
<td>727</td>
</tr>
<tr>
<td>Asia</td>
<td>3,720</td>
</tr>
<tr>
<td>Africa</td>
<td>818</td>
</tr>
<tr>
<td>Oceania</td>
<td>31</td>
</tr>
</tbody>
</table>

Source: Population Reference Bureau

2. **Standardized Test Practice** If 85 of the 170 respondents to a survey answered “yes,” what are the number of degrees for the “yes” part in a circle graph?
   - A 50°  
   - B 68°  
   - C 85°  
   - D 180°  

Angle Relationships

When two lines intersect, they form two pairs of opposite angles called **vertical angles**. Vertical angles have the same measure, so they are **congruent**. Two angles are **complementary** if the sum of their measures is 90°. Two angles are **supplementary** if the sum of their measures is 180°.

<table>
<thead>
<tr>
<th>Angle Relationships</th>
<th>Adjacent Angles</th>
<th>Supplementary Angles</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- **Vertical Angles**
  - \( \angle 1 \equiv \angle 2 \)
  - \( \angle 3 \equiv \angle 4 \)

- **Complementary Angles**
  - \( m\angle 3 + m\angle 4 = 90° \)

**Try These Together**

*Use the diagram at the right to name a pair of angles for each relationship.*

1. adjacent angles
2. vertical angles
3. complementary angles

**Identify the measure of the angle supplementary and the angle complementary to the given angle.**

4. 82°
5. 30°
6. 15°
7. 68°

**Classify each pair of angles as supplementary, complementary, or neither.**

8. 
9. 
10. 

11. **Decorating** Darma is using angles to create a border for a poster. She has made several 35° angles and now wants to draw supplementary angles. Will the supplementary angles be acute, right, obtuse, or straight?

12. **Standardized Test Practice** Angles \( r \) and \( s \) are supplementary. Find \( m\angle r \) if \( m\angle s = 138° \).
   - **A** 38°
   - **B** 42°
   - **C** 48°
   - **D** 35°
You can classify triangles by their angles and sides. The sum of the angles of a triangle is always 180°.

<table>
<thead>
<tr>
<th>Types of Triangles</th>
<th>acute</th>
<th>obtuse</th>
<th>isosceles</th>
<th>equilateral</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>all angles acute</td>
<td>1 obtuse angle</td>
<td>exactly two congruent sides</td>
<td>3 congruent sides</td>
</tr>
<tr>
<td></td>
<td>right</td>
<td>scalene</td>
<td>no congruent sides</td>
<td></td>
</tr>
</tbody>
</table>

**Examples**

**Classify each triangle by its angles and by its sides.**

1. This is a triangle with one right angle, so it is a right triangle. No two sides are congruent, so the triangle is scalene.

2. This is a triangle with all angles acute, so it is an acute triangle. There are two congruent sides, so the triangle is isosceles.

**Try These Together**

**Classify each triangle by its angles and by its sides.**

1.

2.

3.

**HINT:** Refer to the above table to help you classify the triangles.

**Practice**

**Find the missing angle measure for each triangle. Then classify the triangle by its angles.**

4. 15°, 28°
5. 60°, 30°
6. 120°, 36°
7. 72°, 54°
8. 60°, 60°
9. 90°, 25°
10. **Algebra** Find $m\angle E$ in $\triangle CDE$ if $m\angle C = 65°$ and $m\angle D = 58°$.

11. **Standardized Test Practice** A triangle has sides that measure 5 cm, 5 cm, and 8 cm. Classify the triangle by its sides.

   A isosceles   B acute   C scalene   D equilateral
You can classify quadrilaterals by their angles and sides.

<table>
<thead>
<tr>
<th>Types of Quadrilaterals</th>
<th>parallelogram</th>
<th>rectangle</th>
<th>rhombus</th>
<th>square</th>
<th>trapezoid</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>opposite sides parallel and opposite sides congruent</td>
<td>parallelogram with 4 right angles</td>
<td>parallelogram with 4 congruent sides</td>
<td>parallelogram with 4 right angles and 4 congruent sides</td>
<td>exactly one pair of sides parallel</td>
</tr>
</tbody>
</table>

**EXAMPLES**

Classify each quadrilateral by its angles and by its sides.

A. This is a parallelogram with 4 right angles, so it is a rectangle.

B. This is a quadrilateral with exactly one pair of sides parallel, so it is a trapezoid.

**Try These Together**

Classify each quadrilateral by its angles and by its sides.

1. 2. 3.

HINT: Refer to the above table to help you classify the quadrilaterals.

**PRACTICE**

Name every quadrilateral that describes each figure. Then underline the name that best describes the figure.

4. 5. 6.

7. Advertising  Molly is working on a new advertisement for a shoe company. The shoes come in an odd-shaped box. It is a quadrilateral with exactly one pair of opposite sides that are parallel. What name can she use to best describe the box.

8. Standardized Test Practice  Which of the following names cannot be used to describe a square?

   A. trapezoid  B. parallelogram  C. rhombus  D. rectangle

   **Answers:** A, B, C
Two figures are **similar** if their corresponding angles are congruent and their corresponding sides are in proportion.

**Example**

\( \triangle ABC \) is similar to \( \triangle DEF \). What is the missing segment length?

Since the triangles are similar, use a proportion to find the missing segment length.

\[
\frac{AC}{DF} = \frac{BC}{EF} \rightarrow \frac{6}{3} = \frac{10}{x}
\]

Substitute lengths.

\( 6x = 3(10) \) Find the cross products.

\( \frac{6x}{6} = \frac{30}{6} \) Divide each side by 6.

\( x = 5 \)

\( EF = 5 \text{ m} \)

**Try These Together**

**Determine whether each pair of figures is similar. Justify your answer.**

1. 

HINT: Are the angles congruent and the corresponding sides in proportion?

**Practice**

**Find the value of x in each pair of similar figures.**

3. 

4. 

5. **Standardized Test Practice** Which best represents a pair of similar figures?
A tessellation is a repetitive pattern of polygons that fit together with no overlaps or holes. In a tessellation, the sum of the measures of the angles where the vertices of the polygons meet is $360^\circ$.

**Example**

The sum of the measures of the angles of an equilateral triangle is $180^\circ$. Can an equilateral triangle make a tessellation?

Each angle of an equilateral triangle has a measure of $180^\circ / 3$ or $60^\circ$. To find out if an equilateral triangle tessellates, solve $60n = 360$, where $n$ is the number of angles at a vertex.

\[
\frac{60n}{60} = \frac{360}{60}
\]

Divide each side by 60.

\[
n = 6
\]

The solution is a whole number, so an equilateral triangle will make a tessellation.

**Try These Together**

Determine whether each polygon can be used by itself to make a tessellation. The sum of the measures of the angles of each polygon is given.

1. octagon; $1,080^\circ$
2. hexagon; $720^\circ$

Determine whether each polygon can be used by itself to make a tessellation. The sum of the measures of the angles of each polygon is given.

3. triangle; $180^\circ$
4. pentagon; $540^\circ$

Sketch the following tessellations.

5. triangles
6. octagons and squares

7. Computers  Seth wants to make a tessellation to use for a background on a Web page. He would like to use two regular hexagons and one square to form each vertex. Will this work? Why or why not?

8. **Standardized Test Practice**  Which one of the following cannot be used by itself to make a tessellation?
   
   **A** a triangle  
   **B** a square  
   **C** a hexagon  
   **D** a nonagon

---

**Answers:**

1. no  
2. yes  
3. yes  
4. yes  
5. no  
6. yes  
7. No, the sum of the angles of two regular hexagons and one square is $330^\circ$, not $360^\circ$.
8. **D** a nonagon
A **translation** is sliding part of a drawing to another place without turning it.

**EXAMPLE**

The square to the right has had the left side changed. To make sure the pieces, or pattern units, will tessellate, slide or translate the change to the opposite side and copy it.

Now change all of the squares in a tessellation the same way. The tessellation takes on qualities like paintings by M. C. Escher when you add different colors or designs.

**Try This Together**

*Complete the pattern unit for the translation. Then draw the tessellation.*

1. 

   ![Pattern Unit](image)

   *HINT: In the tessellation, all of the pieces must fit together.*

**Practice**

*Complete the pattern unit for each translation. Then draw the tessellation.*

2. 

   ![Pattern Unit](image)

3. 

   ![Pattern Unit](image)

**Standardized Test Practice**

Which shows the completed pattern unit for Figure A?

- A
- B
- C
- D

Answers: 1–3. See Answer Key. 4B
Figures that match exactly when folded in half have **line symmetry**.

**EXAMPLES**

A The figures to the right have line symmetry. Some figures can be folded in more than one way to show symmetry. Each fold line is called a **line of symmetry**.

B You can create figures that have line symmetry by using a **reflection**. A reflection is a type of transformation where a figure is flipped over a line of symmetry.

**Try These Together**

**Draw all lines of symmetry for each figure.**

1.  

2.  

3. 

**HINT:** Think about all of the ways the figures can be folded in half.

**PRACTICE**

4. How many lines of symmetry does a regular hexagon have?

5. **Life Science** Many flowers have lines of symmetry. How many lines of symmetry does a star-shaped flower have?

6. **Standardized Test Practice** Which of the following shows the lines of symmetry in the figure at the right?

   A  

   B  

   C  

   D  

Answers: 1–3. See Answer Key. 4. 6 5 6 D
Chapter 10 Review

Polygon Code

The students in the math club have come up with a secret code for sending messages. The code is based on the number of sides of different polygons, along with addition and subtraction symbols. Use the table below to help you decode the message. Each box is a letter represented by the shaded polygons below it.

<table>
<thead>
<tr>
<th>Number</th>
<th>Letter</th>
<th>Number</th>
<th>Letter</th>
<th>Number</th>
<th>Letter</th>
<th>Number</th>
<th>Letter</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A</td>
<td>8</td>
<td>H</td>
<td>15</td>
<td>O</td>
<td>21</td>
<td>U</td>
</tr>
<tr>
<td>2</td>
<td>B</td>
<td>9</td>
<td>I</td>
<td>16</td>
<td>P</td>
<td>22</td>
<td>V</td>
</tr>
<tr>
<td>3</td>
<td>C</td>
<td>10</td>
<td>J</td>
<td>17</td>
<td>Q</td>
<td>23</td>
<td>W</td>
</tr>
<tr>
<td>4</td>
<td>D</td>
<td>11</td>
<td>K</td>
<td>18</td>
<td>R</td>
<td>24</td>
<td>X</td>
</tr>
<tr>
<td>5</td>
<td>E</td>
<td>12</td>
<td>L</td>
<td>19</td>
<td>S</td>
<td>25</td>
<td>Y</td>
</tr>
<tr>
<td>6</td>
<td>F</td>
<td>13</td>
<td>M</td>
<td>20</td>
<td>T</td>
<td>26</td>
<td>Z</td>
</tr>
<tr>
<td>7</td>
<td>G</td>
<td>14</td>
<td>N</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Answers are located on page 108.
When you find the product of a number times itself, you are finding the square of the number. For example, \( 5 \times 5 = 5^2 \), or 25. Numbers such as 25, 36, and 49 are called perfect squares because they are the squares of whole numbers. The inverse operation to finding a square of a number is finding the square root of a number.

**Square Root**

If \( a^2 = b \), then \( a \) is a square root of \( b \), or \( \sqrt{b} = a \).

There are actually two square roots to the above equation, \( a \) and \( -a \). However, when the symbol \( \sqrt{} \), called a radical sign, is used to represent a square root, it always represents the positive square root.

### Examples

**A** Evaluate \( 9^2 \).

\[
9^2 = 9 \times 9 \quad \text{The exponent tells you how many times the base is used as a factor.}
\]

\[
= 81 
\]

The square of 9 is 81.

**B** Find \( \sqrt{100} \).

Since \( 10^2 = 100 \), \( \sqrt{100} = 10 \).

The square root of 100 is 10.

### Try These Together

1. Evaluate \( 12^2 \).

**HINT:** What is the product of 12 times itself?

2. Find \( \sqrt{49} \).

**HINT:** For which number is 49 the square?

### Practice

**Find the square of each number.**

3. 3  
4. 5  
7. 50  
8. 45  
9. 37  
10. 100

**Find each square root.**

11. \( \sqrt{361} \)  
12. \( \sqrt{484} \)  
13. \( \sqrt{400} \)  
14. \( \sqrt{676} \)  
15. \( \sqrt{1,369} \)  
16. \( \sqrt{1,681} \)  
17. \( \sqrt{3,481} \)  
18. \( \sqrt{160,000} \)

19. **Interior Design** Cole is installing 1-inch square tiles in his entryway. What are the dimensions of the square entryway if he is using 1,296 tiles?

20. **Standardized Test Practice** What is the square of 25?

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>50</td>
<td>625</td>
<td>15,625</td>
</tr>
</tbody>
</table>

You can estimate to find the square root of a number that is not a perfect square.

**EXAMPLE**

Estimate $\sqrt{13}$ to the nearest whole number.

Since 13 is not a perfect square, estimate $\sqrt{13}$ by finding the two perfect squares closest to 13.

$1, 4, 9, 16, 25, \ldots$ List some perfect squares. 13 is between 9 and 16.

$\sqrt{9} < \sqrt{13} < \sqrt{16}$ Find the square root of each number.

$3 < \sqrt{13} < 4$

This means that $\sqrt{13}$ is between 3 and 4. Since 13 is closer to 16 than 9, then the best whole number estimate for $\sqrt{13}$ is 4.

**Try These Together**

Estimate each square root to the nearest whole number.

1. $\sqrt{7}$
   
   HINT: Between which two perfect squares does 7 fall?

2. $\sqrt{48}$
   
   HINT: Between which two perfect squares does 48 fall?

**PRACTICE**

Estimate each square root to the nearest whole number.

3. $\sqrt{75}$
   4. $\sqrt{93}$
   5. $\sqrt{119}$
   6. $\sqrt{150}$
   7. $\sqrt{288}$
   8. $\sqrt{464}$

Use a calculator to find each square root to the nearest tenth.

9. $\sqrt{30}$
   10. $\sqrt{45}$
   11. $\sqrt{63}$
   12. $\sqrt{90}$
   13. $\sqrt{130}$
   14. $\sqrt{333}$
   15. $\sqrt{750}$
   16. $\sqrt{1,122}$

17. **Money Matters** The Etherton family purchased a square lot for their new home that has an area of one acre. An acre is 4,840 square yards. How many yards is one side of their property? Round to the nearest tenth of a yard.

18. **Standardized Test Practice** Find $\sqrt{65}$ to the nearest tenth.

   A 8.0
   B 8.1
   C 9.0
   D 9.1
The Pythagorean Theorem

The longest side of a right triangle is called the **hypotenuse**. The hypotenuse is always opposite the right angle. The other two sides, called **legs**, form the sides of the right angle. Use the **Pythagorean Theorem** to find the lengths of the hypotenuse and the legs of a right triangle.

**Pythagorean Theorem**

**Words:** In a right triangle, the square of the length of the hypotenuse \( c \) equals the sum of the squares of the lengths of the legs \( a \) and \( b \).

**Algebra:** 
\[ c^2 = a^2 + b^2 \]
where \( a \) and \( b \) are the legs and \( c \) is the hypotenuse.

**Example**

A right triangle has legs of 6 cm and 8 cm. What is the length of the hypotenuse?

\[ c^2 = 6^2 + 8^2 \]

Use the Pythagorean Theorem. Replace \( a \) and \( b \) with the values you know.

\[ c^2 = 36 + 64 \]
\[ c^2 = 100 \]
\[ c = \sqrt{100} \]

So, the length of the hypotenuse is 10 cm.

**Try These Together**

Find the missing measure for each right triangle. Round to the nearest tenth.

1. \( a: 17; b: 4 \)
2. \( a: 20; b: 28 \)

**Hint:** Be sure to identify whether a hypotenuse or leg measure is missing before you begin.

**Practice**

Write an equation to solve for \( x \). Then solve. Round to the nearest tenth.

3. \[ \text{5 cm} \]
4. \[ x \text{ ft} \]
5. \[ \text{18 m} \]
6. \[ \text{x mm} \]
7. \[ x \text{ in.} \]
8. \[ \text{6 yd} \]

9. **Construction** Alberto is making a ramp to the door of the chicken coop. The floor of the coop is 14 inches above the ground. The end of the ramp needs to be 3 feet from the coop. How long will the ramp be?

10. **Standardized Test Practice** A rectangle is 12 meters by 9 meters. Find the length of one of its diagonals to the nearest tenth of a meter.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.9 m</td>
<td>15.0 m</td>
<td>21.0 m</td>
<td>225 m</td>
</tr>
</tbody>
</table>

Answers: A, B, C, D
The area \((A)\) of a closed figure is the number of square units needed to cover its surface.

A parallelogram is a quadrilateral with opposite sides parallel and opposite sides congruent. One of its sides is called its base. The length of the segment perpendicular to the base with endpoints on opposite sides is the height.

### Area of Parallelograms

The area \((A)\) of a parallelogram equals the product of its base \((b)\) and height \((h)\).

\[
A = bh
\]

#### EXAMPLES

**A** Find the area of a parallelogram with a base of 10 cm and a height of 4 cm.

- Write the formula for area.
- Replace \(b\) with 10 and \(h\) with 4.
- Multiply.

The area is 40 cm\(^2\).

**B** Find the area of a parallelogram with a base of 13 m and a height of 5 m.

- Write the formula for area.
- Replace \(b\) with 13 and \(h\) with 5.
- Multiply.

The area is 65 m\(^2\).

#### Try These Together

1. Find the area of a parallelogram with a base of 12 in. and a height of 9 in.

   **HINT:** The area of a parallelogram is base times height.

2. Find the area of a parallelogram with a base of 24 ft and a height of 11 ft.

#### Practice

Find the area of each parallelogram.

3. \[
\begin{array}{c}
\text{2 m} \\
\text{5 m}
\end{array}
\]

4. \[
\begin{array}{c}
\text{3 ft} \\
\text{4 ft}
\end{array}
\]

5. \[
\begin{array}{c}
\text{4 cm} \\
\text{6 cm}
\end{array}
\]

6. \[
\begin{array}{c}
\text{3 m} \\
\text{7 m}
\end{array}
\]

7. **Standardized Test Practice** The owner of a video store needs to pave a new parking lot. The parking lot is in the shape of a parallelogram with a base of 80 feet and a height of 120 feet. How many square feet of pavement will he need to order?

   - A 960 ft\(^2\)
   - B 1,600 ft\(^2\)
   - C 9,600 ft\(^2\)
   - D 4,800 ft\(^2\)

Area of Triangles and Trapezoids

You can use the following formulas to find the area of triangles and trapezoids.

<table>
<thead>
<tr>
<th>Area of a Triangle</th>
<th>The area ($A$) of a triangle equals half of the product of its base ($b$) and height ($h$), or $A = \frac{1}{2}bh$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area of a Trapezoid</td>
<td>The area ($A$) of a trapezoid equals half the product of the height ($h$) and the sum of the bases ($b_1 + b_2$), or $A = \frac{1}{2}h(b_1 + b_2)$.</td>
</tr>
</tbody>
</table>

### EXAMPLES

**Find the area of each figure.**

- **A**
  - $A = \frac{1}{2}bh$
  - $A = \frac{1}{2} \times 30 \times 12$
  - $A = 15 \times 12$
  - $A = 180 \text{ cm}^2$

- **B**
  - $A = \frac{1}{2}h(b_1 + b_2)$
  - $A = \frac{1}{2}(6)(8 + 19)$
  - $A = (3)(27)$
  - $A = 81 \text{ in}^2$

### Try These Together

**Find the area of each triangle or trapezoid to the nearest tenth.**

1. base: 4 in.
   height: 9 in.
   HINT: Substitute values carefully.

2. bases: 8 cm, 2 cm
   height: 14 cm
   HINT: Do not forget to add the bases.

### PRACTICE

**Find the area of each triangle or trapezoid to the nearest tenth.**

3. base: 1.2 cm
   height: 1.8 cm

4. base: 23 yd
   height: 8 yd

5. bases: 5 ft, 13 ft
   height: 9 ft

**Find the area of each figure to the nearest tenth.**

6. 

7. 

8. 

9. 

10. 

11. 

12. **Standardized Test Practice** What is the area of a trapezoid with bases of 9 centimeters and 11 centimeters and a height of 4 centimeters?

   **A** 40 cm²  **B** 80 cm²  **C** 160 cm²  **D** 396 cm²
You can use the formula below to find the area of a circle. You can use your calculator for calculations involving \( \pi \).

### Area of a Circle

The area \( (A) \) of a circle equals the product of pi \( (\pi) \) and the square of its radius \( (r) \), or \( A = \pi r^2 \).

#### Examples

**A** Find the area of the circle to the nearest tenth.

\[
A = \pi r^2 \\
A = \pi \times 6^2 \\
A = \pi \times 36 \\
A \approx 113.1 \text{ yd}^2
\]

**B** Find the length of the radius of a circle with an area of 96 m\(^2\).

\[
A = \pi r^2 \\
96 = \pi r^2 \\
\frac{96}{\pi} = \frac{\pi r^2}{\pi} \\
30.6 = r^2 \\
\sqrt{30.6} = r, \text{ so } 5.5 = r
\]

The radius is about 5.5 m.

### Try These Together

**Find the area of each circle to the nearest tenth.**

1. diameter: 5 in.  
2. diameter: 8 m

**Hint:** Use the diameter length to find the radius before you use the area formula.

### Practice

**Find the area of each circle to the nearest tenth.**

3. radius: 19 cm  
4. radius: 11.3 m  
5. radius: 16 yd

6. 

7. 

8. 

**Find the length of the radius of each circle given the following areas. Round to the nearest tenth.**

9. 18 yd\(^2\)  
10. 60 m\(^2\)  
11. 75 m\(^2\)  
12. 23 cm\(^2\)  
13. 48 in\(^2\)  
14. 32 cm\(^2\)

15. **Music** The diameter of a compact disc (CD) is 12 centimeters. The diameter of its hole is 1.5 centimeters. What is the area of one side of a CD?

### Standardized Test Practice

What is the area of a circle with a diameter of 18 meters?

A 2.4 m\(^2\)  
B 5.7 m\(^2\)  
C 254.5 m\(^2\)  
D 1,017.8 m\(^2\)
Complex figures are made of circles, rectangles, squares, and other two-dimensional figures. To find the area of a complex figure, separate it into figures whose areas you know how to find, and then add the areas.

**Example**

**Find the area of the figure.**

The figure can be separated into a rectangle and a triangle.

Find the area of each.

**Area of Rectangle**

\[ A = \ell \times w \]

Area of a rectangle.

\[ A = 12 \times 6 \]

Replace \( \ell \) with 12 and \( w \) with 6.

\[ A = 72 \]

Multiply.

**Area of Triangle**

\[ A = \frac{1}{2}bh \]

Area of a triangle

\[ A = \frac{1}{2}(12)(6) \]

\( b = 12, \ h = 12 - 6 \) or 6

\[ A = 36 \]

Multiply.

The area of the figure is 72 + 36 or 108 square inches.

**Try These Together**

*Find the area of each figure to the nearest tenth.*

1. 2.

**HINT:** Find figures for which you know how to find the area.

3. 4. 5.

**Practice**

*Find the area of each figure to the nearest tenth.*

3. 4. 5.

**Standardized Test Practice**

What is the area of the figure to the nearest tenth?

A 36.4 ft\(^2\)  B 42.3 ft\(^2\)  C 49.8 ft\(^2\)  D 52.1 ft\(^2\)
You can use area models to find the probability of some events.

### Probability

- **Probability**
  - Probability \( P \) is equal to the ratio of the number of ways a certain event can occur to the number of possible outcomes, or
  \[
  P = \frac{\text{number of ways a certain event can occur}}{\text{number of possible outcomes}}.
  \]

### Example

Find the probability that a randomly-dropped counter will fall in the shaded region.

\[
P = \frac{\text{number of ways to land in the targeted region}}{\text{number of ways to land in the entire figure}}
\]

You are comparing two different areas, so you can substitute these areas into the equation.

\[
P = \frac{\text{area of targeted region}}{\text{area of the entire figure}}
\]

\[
P = \frac{6 \text{ square units}}{40 \text{ square units}} \text{, or } \frac{6}{40}
\]

\[
P = \frac{6 \div 2}{40 \div 2} \text{ Divide the numerator and denominator by the GCF.}
\]

\[
P = \frac{3}{20}
\]

### Practice

Find the probability that a randomly-dropped counter will fall in the shaded region.

1. ![Shaded Area](image1.png)
2. ![Shaded Area](image2.png)
3. ![Shaded Area](image3.png)
4. ![Shaded Area](image4.png)
5. ![Shaded Area](image5.png)
6. ![Shaded Area](image6.png)

7. **Standardized Test Practice** A toddler spilled a cup of milk on the floor of a room that has 350 square feet of carpet, and 200 square feet of tile. What is the probability that the toddler spilled the milk on the tile?

   - **A** \( \frac{7}{11} \)
   - **B** \( \frac{3}{8} \)
   - **C** \( \frac{2}{5} \)
   - **D** \( \frac{4}{11} \)
Work Smarter, Not Harder!

Lawanda and the other students in the 4-H club have volunteered with other student organizations to paint the inside of the local youth recreation center. Each club is going to paint a different geometric figure on the wall of the recreation center. Because her group has the fewest members, Lawanda wants to help her club members pick the smallest figure to paint.

Which of the above figures should Lawanda’s club pick? Explain your answer.

Answers are located on page 108.
Three-dimensional figures are called **solids**. You can use a **perspective** drawing to show the three dimensions of a solid in a two-dimensional drawing.

**EXAMPLE**

Make a perspective drawing using the top, side, and front views of the figure below.

**PRACTICE**

**Draw a top, a side, and a front view of each figure.**

1. 2.

**Make a perspective drawing of each figure by using the top, side, and front views as shown. Use isometric dot paper if necessary.**

3. 4.

5. **Standardized Test Practice** What kind of solid has the top view of a circle, the side view of a triangle, and the front view of a triangle?
   - A cone
   - B pyramid
   - C triangular prism
   - D cylinder
A **rectangular prism** is a solid figure that has two parallel and congruent sides, or bases, that are rectangles. The **volume** of a solid figure is the measure of the space it occupies. You can find the volume of a rectangular prism with the following formula.

<table>
<thead>
<tr>
<th>Volume of a Rectangular Prism</th>
<th>Find the volume (V) of a rectangular prism by multiplying the area of the base (B), length (l) times width (w), by the height (h).</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( V = Bh ) or ( V = lwh )</td>
</tr>
</tbody>
</table>

### Example

What is the volume of a rectangular prism with a length of 7 meters, a width of 4 meters, and a height of 10 meters?

\[
V = l \times w \times h \\
V = 7 \times 4 \times 10 \\
V = 280
\]

The volume is 280 cubic meters.

### Practice

**Find the volume of each rectangular prism to the nearest tenth.**

1. \( \text{3 m} \times 2 \text{ m} \times 4 \text{ m} \)
2. \( \text{2.5 cm} \times 7 \text{ cm} \times 1 \text{ cm} \)
3. \( \text{4 cm} \times 4 \text{ cm} \times 4 \text{ cm} \)
4. \( \text{2 mm} \times 6 \text{ mm} \times 2 \text{ mm} \)
5. \( \text{3 m} \times 4 \text{ m} \times 12 \text{ m} \)
6. \( \text{4 m} \times 7 \text{ m} \times 7 \text{ m} \)

7. **Hobbies** The height of a fish tank is 10 inches and the base measures 20 inches by 12 inches. What volume of water can the tank hold when full?

8. **Standardized Test Practice** A 50-pound bag of peanuts is 2 feet by 4 feet by 1 foot. If a 50-cubic-foot space is available for storing the bags, how many can be stored?

**Answers:** 1. 1.2 m³ 2. 1.75 cm³ 3. 6.4 m³ 4. 2.4 m³ 5. 144 in³ 6. 196 m³ 7. 2,400 in³ 8. B
A stack of coins is a model of a cylinder. A cylinder is a solid figure that has two congruent, parallel circles as its bases. Use the formula below to find the volume of a cylinder.

### Volume of a Cylinder

Find the volume \((V)\) of a cylinder by multiplying the area of the base \((\pi r^2)\) by the height \((h)\).

\[
V = Bh \quad \text{or} \quad V = \pi r^2 h
\]

### Example

Find the volume of a cylinder with a diameter of 8 centimeters and a height of 10 centimeters.

- **The diameter of the cylinder is 8 cm. Therefore, the radius is 4 cm.**
- **Estimate:** \(4^2 \times 3 \times 10 = 480\)
- **\(V = \pi r^2 h\)**
- **\(V \approx 3.14 \times 4^2 \times 10\)** Substitute the values for \(\pi, r,\) and \(h.\)
- **\(V \approx 502.4\)**

The cylinder has a volume of about 502 cubic centimeters.

### Try These Together

**Find the volume of each cylinder to the nearest tenth.**

1. Diameter, 2 m; height, 5 m
   - **Hint:** Change the diameter to the radius and then find the area of the base. Multiply the area of the base by the height.

2. Radius, 8 in.; height, 14 in.
   - **Hint:** Find the area of the base and then multiply it by the height.

### Practice

**Find the volume of each cylinder to the nearest tenth.**

3. 6 m 2.5 m
4. 6 ft 17 ft
5. 5.5 m 14 m

6. **Packaging** The diameter of a can of tuna is 3 inches and the height is 2 inches. Find the approximate volume of the can.

7. **Standardized Test Practice** Stella has a can full of water that is 6 cm tall and 8 cm in diameter. She wants to pour the water into a can that is 4 cm in diameter. How tall must the can be?
   - **A** 12 cm
   - **B** 3 cm
   - **C** 24 cm
   - **D** 18 cm

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Surface area is the sum of the areas of all of the outside surfaces of a three-dimensional figure. Use the formula below to find the surface area of a rectangular prism.

**Surface Area of a Rectangular Prism**

The surface area of a rectangular prism equals the sum of the areas of the faces.

\[ S = 2lw + 2lh + 2wh, \text{ where } l = \text{ length}, w = \text{ width}, \text{ and } h = \text{ height}. \]

**EXAMPLE**

Find the surface area of a cardboard carton with a length of 4 feet, a width of 3 feet, and a height of 2 feet.

\[
S = 2lw + 2lh + 2wh \\
S = 2 \times 4 \times 3 + 2 \times 4 \times 2 + 2 \times 3 \times 2 \\
S = 24 + 16 + 12 \\
S = 52
\]

The surface area of the carton is 52 square feet.

**Try These Together**

Find the surface area of each rectangular prism.

1. \[ \ell = 5 \text{ mm}, w = 3 \text{ mm}, h = 2 \text{ mm} \]
2. \[ \ell = 10 \text{ cm}, w = 4 \text{ cm}, h = 6 \text{ cm} \]

**HINT:** Multiply each area you find by 2 to account for the six surfaces of the prism.

**PRACTICE**

Find the surface area of each rectangular prism.

3. \[
\begin{array}{c}
8 \text{ cm} \\
8 \text{ cm} \\
8 \text{ cm}
\end{array}
\]

4. \[
\begin{array}{c}
14 \text{ m} \\
4 \text{ m} \\
6 \text{ m}
\end{array}
\]

5. \[
\begin{array}{c}
4 \text{ in.} \\
3 \text{ in.}
\end{array}
\]

6. **Hobbies** Bob wants to display some of his photographs. Which has more surface area, a 4 inch by 4 inch by 4 inch photo cube, or a 3 inch by 4 inch by 5 inch prism?

7. **Standardized Test Practice** A box is 6 in. by 9 in. by 2 in. How many square inches of wrapping paper would it take to gift wrap this box?

   A 168 in\(^2\)  \quad B 84 in\(^2\)  \quad C 126 in\(^2\)  \quad D 336 in\(^2\)
12-5

Surface Area of Cylinders (pages 538–541)

Use the formula below to find the surface area of a cylinder.

| Surface Area of a Cylinder | The surface area of a cylinder equals the sum of the areas of the circular bases \(2\pi r^2\) and the area of the curved surface \(2\pi rh\).  
\[ S = 2\pi r^2 + 2\pi rh \text{, where } r = \text{radius of the cylinder and } h = \text{height.} \] |

**EXAMPLE**

Find the surface area of a cylindrical drum with a radius of 2 feet, and a height of 5 feet.

\[ S = 2\pi r^2 + 2\pi rh \]
\[ S = 2 \times \pi \times 2^2 + 2 \times \pi \times 2 \times 5 \]
\[ S = 87.96 \text{ Use a calculator.} \]

*The surface area of the drum is about 88 square feet.*

**Try These Together**

Find the surface area of each cylinder to the nearest tenth.

1. \( r = 6 \text{ in.; } h = 10 \text{ in.} \)
2. \( r = 10 \text{ cm; } h = 30 \text{ cm} \)

**HINT:** Remember to add the areas of the bases to the area of the curved surface.

**PRACTICE**

Find the surface area of each cylinder to the nearest tenth.

3. \( \text{3 m} \)
4. \( \text{12 ft} \)
5. \( \text{9 cm} \)

6. **Rocketry** Jule wants to paint his model rocket. The rocket is 28 inches tall and has a radius of 2 inches. He has enough paint to cover an area of 300 in\(^2\). Does he have enough paint to cover his rocket? *Hint:* The top of his rocket tube is an opening for the nosecone, and the bottom is an opening for the motor, so you only have to find the area of the curved surface.

7. **Standardized Test Practice** The diameter of a cylinder is 6 inches and the height is 11 inches. What is the surface area to the nearest square inch?

- A 641 in\(^2\)
- B 471 in\(^2\)
- C 434 in\(^2\)
- D 264 in\(^2\)

*Answers: Answers are calculated using the π key on a calculator and then rounded.*
The precision or exactness of a measurement depends on the unit of measure. The precision unit is the smallest unit on a measuring tool. The smaller the unit, the “more precise” the measurement is.

All measurements are approximate. A more precise method is to use significant digits. Significant digits include all of the digits of a measurement that you know for sure, plus one estimated digit.

**EXAMPLE**

Identify the precision unit of the ruler shown. Then use significant digits to find the measure of the pencil.

The smallest unit is one tenth of a centimeter. The precision unit is 0.1 centimeter. You know for certain that the length is between 17.1 and 17.2 centimeters. One estimate is 17.15 centimeters.

**Try These Together**

1. Identify the precision unit of the ruler below.  
2. Use significant digits to find the measure of the line.

**HINT:** What is the smallest unit of measure?  
**HINT:** Identify two measures that the line is between.

**PRACTICE**

Use significant digits to find the measure of each line.

3.  
4.  
5.  
6.

**7. Standardized Test Practice** Choose the best precision unit for estimating the length of a bedroom.

A 0.1 cm  
B 0.5 in.  
C 0.5 ft.  
D 0.1 mi

Answers: 1. C  
2.6. Sample answers are given: 2.75 cm, 3.125 in, 4.15 cm, 5.375 cm, 6.9375 cm, 7.5 cm, 8.25 cm.
Prize by Volume
Suppose you have just won a raffle. For your prize, you get to fill a container with quarters. You can keep all of the quarters you can fit into the container. Choose from the following containers.

A

B

C

D

Which container would you choose if you wanted to get the most money? Explain how you know.

Answers are located on page 108.
Chapter 1 Review

1. E(xamine)  2. 22  3. 20  4. 13  5. 9
6. 11  7. 15  8. 3

The treasure is MOVIE TICKET.

Lesson 2-1

1. Sample answer:

<table>
<thead>
<tr>
<th>Pets</th>
<th>Tally</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1–2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3–4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5–6</td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>

scale, 1–6; interval, 2

8.

<table>
<thead>
<tr>
<th>Pets</th>
<th>Tally</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1–2</td>
<td></td>
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<tr>
<td>3–4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5–6</td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>

Lesson 2-3

1.

<table>
<thead>
<tr>
<th>Pets</th>
<th>Tally</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pepperoni</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sausage</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vegetable</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mushroom</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Lesson 2-5

1. Stem | Leaf
0      | 5 6
1      | 1 2 7 8
2      | 1 3 4 8 015 = 5

2. Stem | Leaf
0      | 5 6 8 9
1      | 0 6
2      | 0 1 5 5 215 = 25

3. Stem | Leaf
1      | 2 3 4 5 12 = 12
2      | 1 3 6 6

4. Stem | Leaf
0      | 4 6
1      | 2 3 5
2      | 1 4 5 6
3      | 2 6 | 614 = 64

5. Stem | Leaf
0      | 2 3 5 6 7 9 012 = 2
1      | 1 5 6 8 9

6. Stem | Leaf
0      | 2 3 4 5 6 7 9 214 = 24
1      | 1 5
2      | 4 6 8
3      | 3
4      | 1 5
5      | 4 6 8
6      | 4 214 = 24
Answer Key

7. Classical

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>7 8 9</td>
</tr>
<tr>
<td>1</td>
<td>0 1 2 3 3 3 4 5 5 6</td>
</tr>
</tbody>
</table>

710 = 7

Country

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>9 9 9</td>
</tr>
<tr>
<td>1</td>
<td>1 2 2 4 4 5 5 6 6 7</td>
</tr>
</tbody>
</table>

910 = 9

Classical music is priced lower at Annie’s shop.

Lesson 2-6

9.

```
  9 9 9
```

```
  1 0 0 0 2 2 3 3 4 5 6 6 7
```

```
  7 8 9
```

```
  0 1 2 3 3 4 5 5 6 6 7
```

Lesson 2-7

1. 

```
  0 1 2 3 4 5 6 7 8 9
```

```
  3 4 5 6 7 8 9
```

Lesson 2-8

1. 

```
  1 2 3
```

```
  4 5 6
```

```
  0 1 2 3 4 5 6 7 8 9
```

```
  46 00
```

```
  5 0 0 0
```

```
  4 9 0 0
```

```
  4 8 0 0
```

```
  4 7 0 0
```

```
  4 6 0 0
```

```
  4 5 0 0
```

```
  4 4 0 0
```

```
  4 3 0 0
```

```
  4 2 0 0
```

```
  4 1 0 0
```

```
  4 0 0 0
```

```
  3 9 0 0
```

```
  3 8 0 0
```

```
  3 7 0 0
```

```
  3 6 0 0
```

```
  3 5 0 0
```

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  3 4 0 0
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  3 3 0 0
```

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  3 2 0 0
```

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  3 1 0 0
```

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  3 0 0 0
```

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  2 9 0 0
```

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  2 8 0 0
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  2 7 0 0
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  2 6 0 0
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  2 5 0 0
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  2 4 0 0
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  2 3 0 0
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  2 2 0 0
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  2 1 0 0
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  2 0 0 0
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  1 9 0 0
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  1 8 0 0
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  1 7 0 0
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  1 6 0 0
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  1 5 0 0
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  1 4 0 0
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  1 3 0 0
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  1 2 0 0
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  1 1 0 0
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  1 0 0 0
```

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  0 9 0 0
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  0 8 0 0
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  0 7 0 0
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  0 6 0 0
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  0 5 0 0
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  0 4 0 0
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  0 3 0 0
```

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  0 2 0 0
```

```
  0 1 0 0
```

```
  0 0 0 0
```

```
  9 9 9
```

```
  9 9 9
```

```
  9 9 9
```

Lesson 2-7

9.

```
  3 4 5 6 7 8 9
```

Chapter 2 Review

1. 12 2. 73 3. 99

Girls’ lock combination: 12-73-99

4. 13 5. 98 6. 66

Boys’ lock combination: 13-98-66

Chapter 3 Review

1. (4, 4) 2. (−2, −2) 3. (−4, 3)

4. (2, 1) 5. (4, −5) The treasure has coordinates (4, −5).

Lesson 4-5

1. 

```
  y < 10
```

```
  0 1 2 3 4 5 6 7 8 9 10
```

2. 

```
  x > 5
```

```
  0 1 2 3 4 5 6 7 8 9 10
```

3. 

```
  c < 3
```

```
  0 1 2 3 4 5 6 7 8 9 10
```

4. 

```
  j ≤ 8
```

```
  0 1 2 3 4 5 6 7 8 9 10
```

5. 

```
  r < 7
```

```
  0 1 2 3 4 5 6 7 8 9 10
```

6. 

```
  g ≥ 7
```

```
  0 1 2 3 4 5 6 7 8 9 10
```

7. 

```
  x > −1
```

```
  −6 −5 −4 −3 −2 −1 0 1 2 3 4
```

8. 

```
  z ≥ 1
```

```
  0 1 2 3 4 5 6 7 8 9 10
```

9. 

```
  d ≥ 6
```

```
  0 1 2 3 4 5 6 7 8 9 10
```
10. $f \leq 4$

Lesson 4-6

1. $y = 3x$

2. $y = 3x - 5$

3. $y = x + 2$

4. $y = 2x$

5. $y = -4x + 6$

6. $y = 5x + 10$

Chapter 4 Review
1. $A, D$, or $G$  
2. $A$  
3. $D$  
4. $G$

Chapter 5 Review
From least to greatest: \[rac{1}{12}, \frac{3}{10}, \frac{1}{3}, \frac{1}{2}, \frac{3}{4}, \frac{4}{5}, \frac{7}{8}, \frac{19}{20}\]

Chapter 6 Review
1–4. Answers will vary.

Chapter 7 Review
The percent of time dedicated to commercials will vary with the length of the program, the type of program, and the time of the program.

Chapter 8 Review
1. $B$  
2. $B$  
3. $A$  
4. $B$  
5. $A$  
6. $B$

Chapter 9 Review
1. $66$  
2. $220$  
3. $495$  
4. groups of $2$

Lesson 10-1

11. $115^\circ$  
12. $30^\circ$

13. $10^\circ$  
14. $160^\circ$
Lesson 10-2
1c.

Lesson 10-7
5.  

Lesson 10-8
1. 

Lesson 10-9
1.  

Chapter 10 Review
Join the math club.

Chapter 11 Review
The triangle; it has the smallest area, 32 ft², compared to about 50 ft² for the circle and 64 ft² for the square.

Lesson 12-1
1. 

Chapter 12 Review
D; it has a volume of about 452 in³, which is greater than the volume of any of the other containers (384 in³ for A, 240 in³ for B, and about 201 in³ for C).